BETA ANOMALY: AN EX-ANTE TAIL RISK APPROACH

MASTER THESIS

Submitted in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

in Banking and Finance

Univ.-Prof. Dr. Matthias BANK, CFA
Department of Banking and Finance
The University of Innsbruck School of Management

Submitted by
Wolfgang SCHADNER

Innsbruck, June 2018

Affidavit

I hereby declare that this master's thesis has been written only by the undersigned and without any assistance from third parties. I confirm that no sources have been used in the preparation of this thesis other than those indicated in the thesis itself.

This master's thesis has heretofore not been submitted or published elsewhere, neither in its present form, nor in a similar version.

Innsbruck, 28.06.2018

Place, Date Signature

Abstract

This work empirically addresses asset pricing's beta anomaly through a tail risk approach. Building on Merton (1974)'s model of default, ex-ante skewness and kurtosis are directly linked to credit risk. With increasing tail risk, CAPM and Fama & French (1993, 2015) multi-factor models systematically overestimate required stock returns in both absolute and risk adjusted terms. Throughout findings proof, profitability of betting against beta strategies can be significantly enhanced by considering ex-ante tail risk. By developing an alternative asset pricing model accounting for lottery demand and ex-ante tail risk, the beta anomaly can be resolved mostly.

Contents

Abbreviations

ATM	at-the-money
OTM	out-of-the-money
BaB	Betting against Beta
BaB+TR	Betting against Beta and Tail Risk
CAPM	Capital Asset Pricing Model
CF	CAPM extended by lottery demand (FMAX)
CT	CAPM extended by tail risk (TMN)
CTF	CAPM extended by tail risk (TMN) and lottery demand (FMAX)
FF3	Fama-French three factor model
FF5	Fama-French five factor model
FMAX	Lottery demand factor
MAX	Lottery demand coefficient
TMN	Tail risk factor

List of Figures

List of Tables

1 Introduction

Over 40 years, the beta anomaly is a widely known mispricing effect in financial markets. Literature as Haugen & Heins (1975), Frazzini & Pedersen (2014), Schneider et al. (2016) or Baker et al. (2011) show that the empirical security market line within standard asset pricing models is flatter than expected, leading to smaller excess and risk-adjusted stock returns than predicted. Various approaches exist trying to resolve the anomaly. The one used within this empirical study is to release the assumption about normal distributed stock returns in standard CAPM (Sharpe (1964), Lintner (1965), Mossin (1966) by introducing skewness and kurtosis. Given that building on investors' expectations about stocks' future risk-return patterns is fundamental in asset pricing models, skewness and kurtosis coefficients are measured ex-ante from a non-parametric approach following Bali, Hu & Murray (2017). This allows to run a conditional portfolio testing framework of CAPM, Fama & French (1993)'s three (FF3) and Fama & French (2015)'s five factor model (FF5). From Kahnemann & Tversky (1979)'s prospect theory, introducing a decreasing marginal utility function, it can be argued that investors are typically risk averse for (i) negative skewness and (ii) high kurtosis in return series, where I generally refer to these patterns as tail risk. With Merton (1974)'s model it is theoretically shown, how tail risk positively relates to credit risk. Thus, higher tail risk enhances the probability of default causing deteriorating credit risk. At this point, finance theory and empirical research share different opinions. While first one would suggest that investors collect premia for additional (credit) risk taken, studies like Campbell et al. (2008) or Schneider et al. (2016) find existence of a'distress puzzle'. This means, empirically it turns out that stocks facing superior credit risk significantly under-perform their less distressed opposites such that risk-return equilibria are violated.

The found empirical results are in line with (i) Frazzini & Pedersen (2014), providing evidence for the existence of the beta anomaly, (ii) Schneider et al. (2016), confirming that ex-ante skewness positive predicts future returns, and (iii) Campbell et al. (2008), observing the 'distress puzzle'. Furthermore, an alternative asset prizing model based on systematic market risk, lottery demand (cp. Bali, Brown, Murray & Tang (2017) and risk-neutral tail risk is developed and tested.

The reading is set up as follows. Chapter 1 begins with an introduction about relevant literature, theoretical background and evidence of the beta anomaly. Afterwards, two academic approaches are featured about the influence of non-normality onto stock prices. First, theoretical argumentation sourced in Kahnemann & Tversky (1979)'s prospect theory claims that investors are risk averse due to decreasing marginal utility functions. Second, tail risk can be implemented into the Merton (1974) model - with increasing tail risk, probability mass of the asset value's distribution is shifted downwards the debt barrier, thus negative skewness and high kurtosis enhance probability of default causing lifted credit risk. Once the theoretical framework is set, the reading continues at Chapter 2 with initial tests on persistence of skewness and kurtosis as well as empirical evaluation about existence of the beta anomaly. Both, unconditional tests using 1000 random sampled portfolios and conditional tests sorting on rolling beta provide significant evidence for the anomaly to persist in CAPM, FF3 and FF5. Therefore, even the more sophisticated asset pricing frameworks are not able to explain the beta anomaly. For example, lowest beta portfolios realize with 0.05% monthly alpha a 177 basis points higher one than their high beta counterparts, examined under CAPM, within FF5 the gap is slightly smaller but still significant at 1.39% monthly. Further robustness checks of the results is given from substituting beta by ATM implied volatility, hence the beta anomaly can be seen more general as an risk anomaly (cp. Schneider et al. (2016)).

Similar to Schneider et al. (2016), portfolios were then analyzed under conditional forming, monthly re-balancing and double-sorting stocks on rolling beta plus ex-ante skewness/kurtosis. Double-sorting allows to isolate the tail risk anomaly in conditional frameworks. The average difference between neutral and negative skewed portfolios in yearly excess returns is 3.57%, for kurtosis way stronger with 11.04%. Also in terms of risk-adjusted performance measured by regression's alpha, high tail risk portfolios significantly under-perform low tail risk ones. Insights on additional tail risk separation are brought through triple sorting, where the anomaly is even clearer observable. Putting the empirical findings into practical relevance, performance back-tests were made for betting against beta (BaB) and betting against beta plus tail risk (BaB+TR) strategies in the perspective of both long-short and long-only investors.

With significant impact of ex-ante credit risk on stock returns, Schneider et al. (2016)'s statement about persistence of tail risk anomalies even after taking into account lottery demand, an alternative asset pricing model is developed based on standard CAPM's beta, Bali, Brown, Murray & Tang (2017)'s 'FMAX' factor representing lottery demand and a self created 'TMN' factor capturing ex-ante tail risk. This model is named as 'CTF'. The CTF model turns out to realize high coefficient significance, equally R^2_{adj} as FF5 and a superior deletion of the beta anomaly. After various robustness test, the risk anomaly quantified as the absolute correlation between regression's alpha and the systematic market risk exposure ($cor(\alpha,\beta)$) is reduced from 0.80 in CAPM over 0.62 in FF5 down to 0.17 in CTF. Thus, accounting for tail risk resolves most of the beta anomaly.

2 Theoretical Background

Let the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Mossin (1966) and Lintner (1965) be the starting point of this work. When published in the 1960ies, CAPM became the pioneering cornerstone for the entire world of finance. With its fundamental mechanism, that financial asset pricing is always a trade-off between expected return and risk measured relative to market variance, CAPM plays ever since one of the most dominant roles in both financial research and investor's practice. Nonetheless its wide use, criticism came very soon once it was published. One of the first prominent works uncovering CAPM's inefficiencies is the paper by Black et al. (1972), stating that stocks' expected returns are not necessarily proportional to their beta value, hence they reject the pure form of CAPM arguing that only two factors, mean and systematic variance, are not sufficient to correctly describe asset pricing behavior. Black (1972) further describes the beta related mispricing as a source of leverage constraints by first delivering evidence that the empirical security market line is flatter than suggested under CAPM. From these early critics, many more followed over time and finance research still puts high effort upon explaining anomalies arising in CAPM based asset pricing models. With Black et al. (1972)'s opening discussion, research generally followed two different approaches for asset pricing modeling. First - and more popular - approach was to add factors to the originally CAPM which are believed to be economically rational and have explanatory capability. Prominent representatives of this approach are Fama & French (1993, 2015) with their three and five factor models. Others used a rather statistical point of view with their opinion, that CAPM only captures the first and second moment (mean and variance, respectively) of a stock's return distribution, while it is empirically proven that realized return distributions mostly differ from normal assumptions, thus higher distributional moments such as skewness (3rd moment) and kurtosis (4th moment) do exist and must have an influence on asset prices. Whether to use a multi-factor approach or a multi-moment approach seems to be a matter of attitude, nevertheless Fama & French (1993, 2015) multi-factor models established themselves as probably the most popular used models, perhaps due to its easy interpret-ability and economical reasoning. That multi-moment models were not able to find that broad acceptance may arises out of the difficulty to measure distribution's higher moments. Since in earlier works the only way to estimate skewness and kurtosis was out of historical return series, the main problem in multi-moment models may occurred out of the many assumptions to be set and the corresponding measurement sensitivity. In addition, multi-moment models always were - by construction - backward looking, which causes severe misestimation of stock returns when model parameters (skewness and kurtosis) vary strongly over time. With new research insights, connecting return distributions with corresponding option implied volatility surfaces, skewness and kurtosis can be estimated ex-ante, allowing multi-moment asset pricing models to become forward looking rather than relying on historical distributions. That this risk neutral pricing approach again encourages research in asset pricing with skewness and kurtosis is represented for example by Bali, Hu & Murray (2017) or Schneider et al. (2016). To point out the economical relevance of tail risk and to reduce bias, empirical results are tested under CAPM, Fama & French (1993) three factor and (FF3) Fama & French (2015) five factor (FF5) settings. Thus, the discussion of whether to prefer multi-factor models or not, does not softener the evidence, that tail risk is not sufficiently considered by popular asset pricing models.

2.1 The Low Beta Anomaly

It is an old story, CAPM was published (Lintner (1965), Mossin (1966), Sharpe (1964)) and ever since finance researcher showed that the true security market line is less steep than proposed by CAPM. Since theoretical and empirical security market lines deviate from each other, the main question arises, if this deviation is really an anomaly in risk-return trade-off so that there is 'free lunch' available, or is it solely due to the fact, that not all risk components of stock returns are captured by the model.

Empricial evidence for the existence of the beta anomaly is broad. No matter whether one follows earlier studies such as Jensen (1968), Dybvig & Ross (1985) ,Fama & French (1996) or Jagannathan & Wang (1996) building their research on CAPM's unconditional form where beta's are not considered to be time varying, nor conditional CAPM tests (e.g. Lewellen & Nagel (2006) or Baker et al. (2013)). The beta anomaly exists, verified in the U.S. market for the past 90 years (cp. Haugen & Heins (1972)). Haugen & Baker (2012) show that the beta anomaly does not only have a long time of presence, nor is it limited solely to the U.S. market, it is rather a global phenomena. Even since the detection of the anomaly 45 years ago (cp. Black et al. (1972)) it still dominates financial markets. While Haugen & Baker (2012) uncover evidence for the beta anomaly to be a worldwide phenomena, Baker et al. (2014) provide insights of increasing profitability of betting against beta strategies when investors additionally distinguish between low and high beta countries.

As there exist many studies confirming the anomaly, there are several explanations. One possible reason could be institutional investors' borrowing and leverage constrains (cp. Gibbson (1982), Kandel (1984), Shanken (1985), Frazzini & Pedersen (2014)). For example, some market participants such as mutual funds are restricted in leveraging and short selling. In order to beat their benchmarks in absolute performance they have to bet on high beta stocks as they are not allowed to leverage the risk-efficient market portfolio, which leads to an over proportional high demand in high beta stocks causing under-performance. That this approach is also an explanation of the anomaly to remain is investigated by Li et al. (2014) who point out that considering the large costs of short-selling high beta stocks makes it unprofitable for hedge funds to bet on that anomaly through beta-neutral portfolios, long in low-beta stocks and short in high-beta stocks. Blitz & van Vliet (2007) and Blitz & van Vliet (2011) see besides leverage constraints also behavioral biases and benchmarking problems as the reason for the low risk anomaly to continue.

Other researchers as Falkenstein (1994) and Ang et al. (2006) find reasons in unsystematic return variance as an additional source for CAPM anomalies. Johnson (2005) further see, that especially expected returns of levered firms should decrease with higher levels of idiosyncratic volatility. This relation is confirmed by Boehme et al. (2009), who observe possibilities in the theoretically diversify-able idiosyncratic risk to empirically occur due to the enormous efforts of short selling those stocks. From literature going into that direction, Schneider et al. (2016) build their empirical evaluations upon both systematic (beta) and option-implied volatility measures to find persistance in the anomaly under both risk proxies.

Importantly, besides others, Bali et al. (2011, 2014) find that lottery demand influences the slope of the security market line (SML), such that with increasing volatility, the empirical SML is not only flatter but also downward sloping. By accounting for lottery demand via their FMAX factor, they believe the beta anomaly to resolve.

2.2 Non-Normality effects

The importance of return distribution's higher moments was first described by Rubinstein (1973) and empirically shown by Kraus & Litzenberger (1976). Generally, I find two economic argumentations why risk-adjusted returns are different if stock return distributions are negatively skewed and/or leptokurtic. Literature such as Bakshi et al. (2003), Kelly & Jiang (2014) or Konno et al. (1993) find favor for tail risk compensation as a natural source when applying Kahnemann & Tversky (1979) prospect theory. Another argumentation why investors should consider tail risk can be made via the Merton (1974) model of default. The two argumentations are described in more detailed below. However, measuring skewness and kurtosis ex-ante from option data, recent literature finds different empirical relations between equity returns and tail risk.

There are Bali (2009), Xing (2010), Rehmann & Vilkov (2012) or Schneider et al. (2016) who find positive relation between ex-ante skewness and risk adjusted returns while others like Conrad et al. (2013) find the opposite relation.

From literature research I find two popular approaches measuring ex-ante moments. One is the model-based and widely used Bakshi et al. (2003) ('BKM') approach of estimating stocks' implied moments by regression analysis of option prices over different maturities, papers using this model are besides others Bali & Murray (2013), Bali (2009), Bali et al. (2014), Xing (2010) or Conrad et al. (2013). Second approach in empirical finance literature are non-parametric proxies, estimating ex-ante moments directly from the implied volatility surface. This approach is represented by Mixon (2010), Bates (2000), Cremers et al. (2008), Alexander & Veronesi (2009) or Bali, Hu & Murray (2017). Liu & van der Heijden (2015) gives a detailed comparison of currently used risk-neutral skewness and kurtosis measures for both, model-based and non-parametric settings. Supporting the non-paramteric approach, Bali, Hu & Murray (2017) derive equall results within their analysis under Bakshi et al. (2003) measure and non-parametric proxy. Mixon (2010) does not only give an overview among different non-parametric skewness/kurtosis estimations, he also points out reasons to prefer them above the BKM model. Importantly, in the work of Bali, Hu & Murray (2017) they highlight key characteristics of demand based option pricing (cp. Bollen & Whaley (2004) and Garleanu et al. (2009)) which causes non-linearity in options' volatility surfaces and allows the comparison between model-based and non-parametric estimations of risk neutral moments.

Remarkably, simultaneously to conditional CAPM building on time varying rolling betas, Harvey & Siddique (2000) point out the importance of building analysis upon conditional skewness, allowing for time dependent variation in this measure. Furthermore, Lin & Liu (2017) focus on lottery demand effects quantified by using Frazzini & Pedersen (2014)'s MAX measure in combination with return skewness, which is close to the empirical survey also putting attention on CAPM extensions by lottery-demand and tail risk.

Utility Theory and Tail Risk. Kraus & Litzenberger (1976) were one of the first to extend standard CAPM by the third moment (skewness). Their starting point is investors' utility maximization of Markowitz (1952)' mean-variance portfolio theory. Extending the quadratic utility function (mean-variance) by a cubic solution, Kraus & Litzenberger (1976) provide a first asset pricing model which is able to account for skewness in return distributions. Notably, in their model investors are averse among negative skewed return distributions and in favor for positive skewness. Their asset pricing model is a simple extension of CAPM by a skewness term:

$$R_i - R_f = b_1 \cdot \beta_i + b_2 \cdot \gamma_i, \tag{1}$$

where β_i and γ_i represent the stock's systematic volatility and skewness respectively. Harvey & Siddique (2000)) start at the same point as Kraus & Litzenberger (1976) with the idea, that CAPM fails due to its disability in capturing skewness risk. Very fundamental in asset pricing theories is the one-period investor's first-order condition to take on risky positions:

$$E[(1+R_{i,t+1})m_{t+1}|\Omega_t)] = 1, \tag{2}$$

given $R_{(i,t+1)}$ represents return on risky asset i for the next period $t+1$ with the marginal rate of substitution $m_{(t+1)}$ under all information available Ω_t in the current period. The marginal rate of substitution is what literature often refers as the pricing kernel or the stochastic process for discounting the asset's future payoffs. Most asset pricing models adapting for skewness try to do so by defining different forms of the marginal rate of substitution. For standard CAPM, the marginal rate of substitution is assumed to be linear:

$$m_{t+1} = a_t + b_t R_{M,t+1}, \tag{3}$$

with $R_{(m,t+1)}$ as the expected market return for the next period and a_t as well as b_t reflecting the available information set Ω_t at the current period. Resolving for the expectations, equation 2 can be rewritten as

$$E_t[1+R_{i,t+1}] \cdot E_t[m_{t+1}] + cov_t[(1+R_{i,t+1}), m_{t+1}] = 1, \tag{4}$$

such that the general form of an asset's expected return is given by

$$E_t[1+R_{i,t+1}] = \frac{1}{E_t[m_{t+1}]} - \frac{cov_t[(1+R_{i,t+1}), m_{t+1}]}{E_t[m_{t+1}]}. \tag{5}$$

Assuming that Kahnemann & Tversky (1979) prospect theory holds, then an investor's utility function will be S-shaped. This means, that marginal utility of a gain is decreasing over size, while the marginal disutility of a loss is also shrinking among severity of the loss. Prospect theory further states a greater slope on the loss side of the utility function than on the gain side, thus a loss of a given size exposes the investor with more disutility than a gain for the same amount. For example, one possible notation of this theory is the form of power-log utility functions describing investor's preferences as follows:

$$\begin{aligned} U &= ln(1+r_e) \quad \text{for } r_e \geq 0 \\ U &= \frac{1}{\gamma}(1+r_e)^{\gamma} \quad \text{for } r_e \leq 0 \end{aligned} \tag{6}$$

with r_e as the return on equity and γ as the power capturing risk aversion, where γ is less or equal to zero (cp. Kale & Sheth (2015)). With this difference in gain utility and loss disutility, skewness and kurtosis have crucial impact. Assume the following setting, there are two firms, A and B. Both firms are identical in expected return on equity and beta, but A's return distribution is expected to be normal while B faces kurtosis. Linear utility functions would treat them as equal since both deliver the same expected return. Under power-log utility functions, firm B's higher probability mass in the tails induces lower marginal utility, hence investors are naturally averse among kurtosis preferring firm A above B. Applying $\gamma < 0$, negative skewness in firm B would balance probabilities on the negative side, but as losses count heavier than gains, investors are averse regarding negative skewness.

Following prospect theory (cp. Kahnemann & Tversky (1979)), it can be shown that (i) investors should indeed care about non-normality in stocks' return distributions and (ii) CAPM, besides other multi-factor asset pricing models, is not able to capture this preference effect. Prospect theory does not necessarily support the idea that non-normal return distributions imply additional risk components, it is rather a way of arguing in utilities differing among several distribution shapes. Given same expected return and systematic risk, prospect theory shows that the distribution's shape impacts the investor's expected utility. Hence investors should prefer positive skewed firms with low kurtosis and require a premium for firms with higher kurtosis and negative skewness. That this statement holds is provided by Konno et al. (1993). With a focus on skewness, they use a very similar utility theory related argumentation to show that between two portfolios with equal mean and variance, investors should have a strong preference for the portfolio with larger (positive) skewness. Konno et al. (1993) hence deliver beneath other studies evidence, that neither Markowitz (1952) nor CAPM are sufficient to provide precise asset pricing.

Bakshi et al. (2003) find general laws to connect utility functions with risk neutral moments. They show, that ex-ante skewness is a natural phenomenon as a source of investors' aversion among tail risk. They further highlight the important role of excess kurtosis in reflecting investors' risk aversion. As conclusion, Bakshi et al. (2003) retrieve that risk-neutral skewness is a consequence of risk aversion - as suggested in prospect theory (cp. Kahnemann & Tversky (1979)) - and leptokurtic return distributions.

Tail Risk drives Credit Risk. Not only utility theory delivers a solution for skewness and kurtosis related mispricing, Schneider et al. (2016) also connect the effects of higher moments to credit risk. Following their argumentation, with increasing credit risk, CAPM and other multifactor models overestimate the real market risk such that (model) expected returns are significantly higher than realized returns. Their approach is based on the basic Merton (1974) credit risk model, where a firm's equity can be seen as kind of a European call option. If a firm's asset value falls bellow a certain default level, the firm will be unable to repay its debt hence has to declare default. In the Merton (1974) model, this default level would equal the strike and asset value A_t the underlying of a European call option, if we assume for now that debt can have a certain face value D, the variation of asset value can be isolated solely due to fluctuations in equity E. As E dominates asset value, the default level can also be drawn in the return distribution (see Fig. 1). If at maturity T the return is above that strike level, then the investor can expect a payoff, in the case the firm's return until maturity falls below the default level such that the asset value will be below the outstanding debt ($A_T < D$), the payoff to the investor will be zero. Hence equity of a firm can be seen as an European option, where $E_T = max(A_T - D, 0)$. The credit risk within this model will be the probability that the firm value will fall below the default level ($Pr(A_T < D)$) at maturity.

$$A_T = E_T + D_T \tag{7}$$

$$A_T = max(A_T - D, 0) + D_T \tag{8}$$

A basic assumption within that model is that the asset value A_t follows an geometric Brownian motion

$$dA_t = \mu A_t d_t + \sigma A_t dW_t, \tag{9}$$

with μ as the mean return on assets and σ the asset's volatility. Considering that debt is assumed to be one single outstanding bond with defined face value, μ and σ are determined by the development of equity value with the basic assumption, that equity returns are normal distributed and respectively log-normal distributed asset value. At maturity T, the probability of default can be visualized within the equity return distribution plot as the area below the probability distribution left to the default level, see Fig. 1.

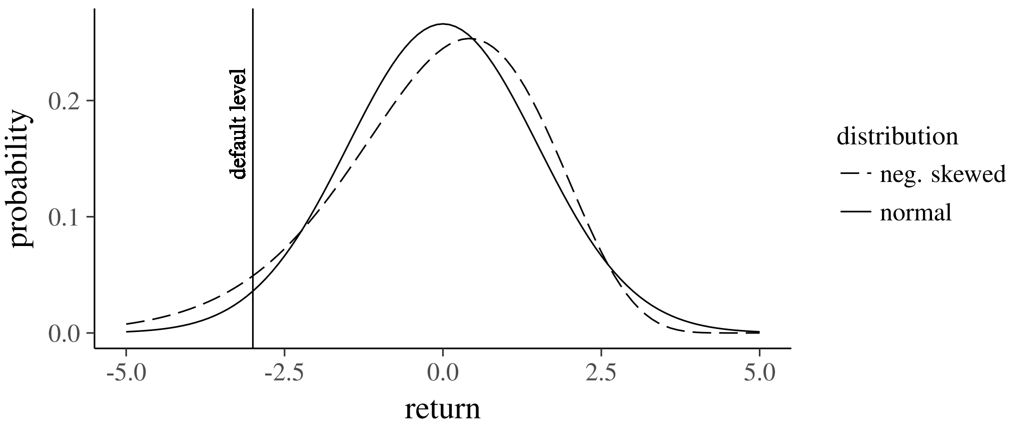

Figure (1) Skewness effect on credit risk. Two return distributions being identical in $E[r_e]$ and σ but with distinctive skewness, thus credit risk ($Pr(A_T < D)$) is on different levels.

How skewness and kurtosis come into play with credit risk can be best explained through this plot. The more a distribution turns towards negative skewness, the more probability mass will be shifted into the area below the default level (area left to the default level), thus credit risk as determined in the Merton (1974) framework increases. The same effect arises for increasing kurtosis. The higher the probability to fall below the default level, the higher the credit risk, hence negative skewness and leptokurtosis are directly pushing probability of default. So for two comparable stocks being equal in expected return and beta, their risk levels can still be distinguished if they differ in higher moments of their return distributions. Thus, not only from utility theory argumentation, also through the credit risk channel investors have a rational reason to trade on different tail risk expectations. By this credit risk channel it is theoretically shown, that tail risk induces an additional risk source which cannot be captured by CAPM or Fama & French (1993, 2015) models.

Following the credit risk channel, if investors do not require compensation for that additional risk source and avoid buying them, stocks with low tail risk will outperform those with higher tail risk. This statement is confirmed by studies from Cremers & Weinbaum (2010), DeMiguel et al. (2013) or Schneider et al. (2016) and further supported within this work. From empirical analysis it seems, that for non-normal return expectations, CAPM systematically overestimates market risk which (mis-)leads into too high return expectations. Taking into account that ex-ante skewness and kurtosis do significantly deviate from normal assumptions, I see this as a natural source for CAPM's overestimation of expected stock returns. Common asset pricing models empirically fail not only to correctly compensate for volatility (2nd moment)(= beta anomaly) but also for skewness (3rd moment) and kurtosis (4th moment)(= tail risk anomaly).

2.3 About systematic and unsystematic moments

Markowitz (1952)' portfolio theory is clearly the base in modern finance. This first asset pricing model sets an asset's expected profitability into relation to its overall return variance. Sharpe (1964), Mossin (1966) and Lintner (1965) then further improve this two parameter model and build the CAPM where they show, that due to diversification effects the expected return is proportional to the systematic market risk rather than to total variance. With CAPM, asset pricing now expresses expected returns relatively to systematic risk exposure. Following the birth of CAPM, extensions by further moments such as Kraus & Litzenberger (1976) followed, whether the higher moments are taken under total measure analogously to the variance in Markowitz (1952) portfolio theory or systematic (e.g. Kraus & Litzenberger (1976)) as in CAPM. Literature thus comes to different opinions, whether to account for total, systematic or unsystematic measurement of higher moments.

Several previous studies (such as Kraus & Litzenberger (1976), Harvey & Siddique (2000) or Schneider et al. (2016)) follow the idea of systematic risk of higher moments. Just as the systematic variance in CAPM, skewness and kurtosis are thought to be decomposable into systematic and idiosyncratic parts, where through portfolio diversification investors can eliminate the unsystematic portion so that only systematic moments remain. In these models, investors only require compensation for systematic skewness and kurtosis. This opinon is the most widely one found in finance literature. Different to those findings are the studies of Chabi-Yo (2012), Xing (2010), Rehmann & Vilkov (2012) or Bali, Hu & Murray (2017). Chabi-Yo (2012) find that systematic skewness does not have significant predictive power for expected stock returns, rather is the unsystematic component of skewness from importance as the systematic part is restricted due to investor preferences. Bali, Hu & Murray (2017) also find support for the hypothesis, that when it comes to skewness and kurtosis, the unsystematic components are of value rather than systematic parts. Their results are built upon the argumentation that skewness and kurtosis are driven by firm-specific information such that investors build bets in the option markets directly influencing the option implied ex-ante measures. From their suggestions, the relevance of systematic skewness and kurtosis disappears when control parameters are included in the regression analysis whereas unsystematic components still show positive (negative) relation between risk-neutral skewness (kurtosis) and expected returns. There are also Bollen & Whaley (2004) and Garleanu et al. (2009) showing through their demand-based option pricing models that firm-specific information drives option prices to reflect unsystematic skewness and kurtosis.

Besides testing the impact of total risk-neutral moments computed under both BKM and non-parametric approach, Bali, Hu & Murray (2017) decompose the total ex-ante skewness (kurtosis) into idosyncratic and systematic components methodology suggested by Bakshi et al. (2003):

$$Skew_i = \frac{\beta_{RN,i}^3 \sigma_m^3}{\sigma_i^3} \cdot Skew_m + \frac{\sigma_{\varepsilon,i}^3}{\sigma_i^3} \cdot Skew_{\varepsilon,i}, \tag{10}$$

where $Skew_i$ ($Skew_m$) denotes the total risk-neutral skewness of a firm i (the market m) and $\beta_{RN,i}$ the risk-neutral expectations for the firms market risk loading. Building upon this equation, a firm's systematic risk-neutral skewness may be calculated using

$$Skew_{S,i} = \frac{\beta_{RN,i}^3 \sigma_{RN,m}^{3/2}}{\sigma_{RN,i}^{3/2}} \cdot Skew_m, \tag{11}$$

so the unsystematic risk-neutral skewness will be

$$Skew_{U,i} = Skew_i - Skew_{S,i}. \tag{12}$$

Keeping the same index notation, the risk neutral kurtosis can be analogously separated with

$$Kurt_i = \frac{\beta_{RN,i}^4 \sigma_{RN,m}^4}{\sigma_{RN,i}^4} \cdot Kurt_m + \frac{\sigma_{\varepsilon,i}^4}{\sigma_i^4} \cdot Kurt_{\varepsilon,i} \tag{13}$$

as the overall risk-neutral kurtosis and

$$Kurt_{S,i} = \frac{\beta_{RN,i}^4 \sigma_{RN,m}^2}{\sigma_{RN,i}^2} \cdot Kurt_m, \tag{14}$$

representing the systematic risk-neutral kurtosis. The idiosyncratic risk-neutral kurtosis is again simply the difference between the total ex-ante kurtosis minus the systematic component

$$Kurt_{U,i} = Kurt_i - Kurt_{S,i}. \tag{15}$$

3 Data and Modeling

Aim of this work is to examine whether the impact of ex-ante distribution's higher moments on risk-return trade-offs varies among the beta anomaly. As stated in previous sections, a risk-neutral approach is used generating advantages of (i) taking investor's expectations of future returns rather than relying on historical data and (ii) avoiding additional assumptions about look-back period and compounding frequency of returns when determining the distribution, bringing the overall empirical analysis closer to real world market behavior. Thus, implied ex-ante measures derived from option prices should deliver more precise research but comes with the drawback, that for every stock frequently traded option data has to be available. Through this restriction, stocks without any related options are excluded from the survey, which may induces bias potential in the evaluation. However, as also discussed in other works (e.g. Bali, Hu & Murray (2017)), an ex-ante approach should deliver the clearest picture of risk-return trade-offs that come closest to real world relations as asset pricing models themselves are, by definition, mechanisms to quantify investor expectations about stock performance. Therefore, determining a stock's distribution by using realized returns prior to the actual moment of pricing the asset does thus violate the basic assumption stated in the definition of asset pricing models. Bali, Hu & Murray (2017) further criticizes that current research relies heavily on that backward-looking bias connecting historical risk with future return data. In order to avoid this violation, I build the empirical work in this paper based on a risk-neutral approach comparable to Schneider et al. (2016), Bali, Hu & Murray (2017) or Bali (2009).

Risk-neutral asset pricing, however, seems to be more persistent with real world behavior but comes with more difficulty to implement. While historical return data are broadly available, capturing investors' ex-ante expectations about future risk and return distributions is more challenging. Therefore, I follow several recent studies using option prices to derive the underlying stock's characteristics. Studies among demand based option pricing models such as Bollen & Whaley (2004), Garleanu et al. (2009) or Bali, Hu & Murray (2017) show that the relation between a stock's call prices to its put prices over different maturities and/or moneyness contains information about implied moments like volatility, skewness and kurtosis. Demand based option pricing argues that stock return expectations have to be reflected within option prices. When positive returns are expected, investors demand for call options and try to sell puts, such that call prices rise and put prices fall respectively. Given options with certain strikes, time to maturities and underlying prices, option pricing formulas (e.g. Bachelier (1900), Black & Scholes (1973) or Hagan et al. (2002)) imply a level of volatility that satisfies the current option price which is set by the market. Fulfilling believes about normal distributed returns, the implied volatility has to be flat among strike prices. This is the case when all call options are traded for the same price as the put options with equal degree of moneyness / sensitivity to the underlying (delta). Nonetheless, most equity options cannot satisfy these characteristics such that for the same level of delta, the call price differs to the respective put price. This effect causes option's implied volatility to turn from a flat to a smile shape. Dependent on how the put-call-parity varies across strike prices, the implied volatility function is either symmetric (skewed) or turned in the point of the underlying price (skewed and leptokurtic). From this argumentation follows, observing a stock's put-call parity with different strike levels, one can conclude about the stock's implied volatility, skewness and kurtosis. Empirical evidence proving this relationship is broadly available, e.g. An et al. (2014), Rehmann & Vilkov (2012), Xing (2010), Bali (2009), Bali, Hu & Murray (2017) and DeMiguel et al. (2013).

That CAPM's normal assumption about equity returns does not hold is well known. Older studies such as Kraus & Litzenberger (1976), Friend & Westerfield (1980), Rubinstein (1973) or Kane (1982) already show that besides mean and variance, higher statistical moments of return distributions are valuable in asset pricing. Since then, asset pricing models were split into two categories: models implementing higher moments (e.g. Kraus & Litzenberger (1976), see Section 2.2) and models introducing additional parameters (e.g. Fama & French (1993, 2015)). While Fama-French (Fama & French (1993, 2015)) multifactor models were able to increasingly rise popularity over time, Schneider et al. (2016) argue that return-distribution related models even have stronger predictive power, with the result, that supplementary factors like HML or SMB from the Fama & French (2015) models ares systematically unable to capture skewness effects.

3.1 Measuring Risk Neutral Moments

To fulfill the ex-ante criteria, risk-neutral return distributions are derived from option prices. At first sight, this task seems quite straight forward, however, given the existence of different option pricing models, literature additionally delivers a variety of approaches measuring risk-neutral moments out of them.

Non-Parametric Approaches. Xing (2010) and Bali, Hu & Murray (2017) argue that the implied volatility surface of stock options has explanatory abilities. Brought in Section 2, demand based option pricing shows that inequalities in the put/call parities cause non-constant implied volatility surfaces among option prices. Thus Mixon (2010) derives a non-parametric approach by observing the smile and angle of the implied volatility surface, where the smile is the indicator of implied skewness and the angle of the volatility function in the point of the ATM implied volatility is the proxy for the implied kurtosis. Hence making use of Mixon (2010)'s approach, the risk-neutral distribution's characteristics can be estimated as follows:

$$Skew^{NonPar} = CIV_{25} - PIV_{25} \tag{16}$$

$$Kurt^{NonPar} = CIV_{25} + PIV_{25} - CIV_{50} - PIV_{50} \tag{17}$$

with CIV_{25} (PIV_{25}) as the implied volatility level for a 25% out-of-the-money call (put) and CIV_{50} (PIV_{50}) 50% OTM respectively.

That those proxies are valid measurements and deliver identical results as the model-based regressions is supported by Bali, Hu & Murray (2017). The two non-parametric approximations are the implemented estimation method for risk-neutral skewness ($Skew^{NonPar}$) and kurtosis ($Kurt^{NonPar}$) within the empirical part of this survey here.

BKM approach and the CBOE indices. In terms of completeness, also the typical model-based approach by Bakshi et al. (2003) (in the following named BKM) is explained here. Through implementing differential option pricing formulas, they capture stochastic evolving ex-ante volatility, skewness and kurtosis. They were one of the first to introduce an estimation tool for the risk-neutral return distribution building on option data and further proofing, that stock return distributions indeed relate to their risk-neutral skewness estimation. The BKM model is built on several assumptions. First, that strike prices are available on a continuous basis ranging from zero to infinity and second assumption, that options are of European-style. While for the first assumption literature delivers several methods to deal with the problem, which could be either using raw data or interpolating the missing data points with smoothing splines. The second assumption, as Liu & van der Heijden (2015)) argue, is more difficult to relax as most stock options are of American-style rather than of European-style. That this approach is quite popular among researchers can be seen in studies like Mayhew (2002), Han (2008), Conrad et al. (2013) or Bali et al. (2014).

Another approach for determining implied moments is brought by the Chicago Board of Exchange (CBOE). Introduced in 1993, the CBOE provides the popular volatility index (VIX, currently VOX) which is built using option implied volatilities estimated out of ATM option data. The VIX can be seen as a milestone in finance practice as it was the first forward looking volatility measure, providing the base for further risk-neutral related research. The model-free concept of the current VIX is given by Demeterfi et al. (1999). Almost 20 years later in 2011, the CBOE published its first risk-neutral measure for skewness (SKEW), which became a benchmark for the S&P 500 forward looking distributional risk. So far, the CBOE does not yet provide a similar measure for risk-neutral kurtosis, Liu & van der Heijden (2015) shows a BKM equivalent approach to construct a kurtosis estimator based on the CBOE's SKEW methodology.

3.2 Other settings

Besides paramter definition for risk-neutral moments, additional inputs and variable settings used in this study are defined as follows.

Input Data. All equity and option data used are derived from the data source Bloomberg L.P. The stock universe is determined by all accessible U.S. stocks for the given time horizon of January 2005 to March 2018 with the restriction, that frequently traded options upon the stock's equity need to be available. The starting date of the observation data is set due to the reason of data availability, the initial date of the data source for providing information among option implied volatility starts with January 2005, thus, this survey also begins with that date. To avoid upward bias, the dataset takes into account liquidation and bankruptcy of companies such that it is corrected for survivorship bias. Additionally, ensuring that effects occurring from nano-cap firms are removed, a minimum market capitalization level of 300 Mio. USD is introduced as a stock's requirement in order to be considered within the survey. This market capitalization barrier is checked on a monthly basis, firms can fall out or come back in during the time whether their current market capitalization for that month is fulfilling the level or not. Accounting for all this criteria, the subset of the U.S. equity market consists of overall 3651 stocks. To remove survivorship bias, I also include stocks which do not survive until the end of the data time horizon(=1st Mar 2018), under the general condition, that at every rebalancing date, a stock's market capitalization has to be above the 300 Mio USD level to be considered in the analysis for the following month. Anyway, as I always build portfolios using value weighting, firms with small market capitalization will contribute less to a portfolio's return than a large cap firm does, thus small sized firms have weaker influence on the empirical evaluation.

SMB, HML, RMW and CWA are referred to as the Fama-French factors, the data for those values as well as the CRSP market index is sourced from K. French's data library[1].

Market index. Given that this work aims to represent the entire U.S. equity market, but has to restrict the data on given criteria (e.g. option availability), I see two market indexes as plausible benchmarks. The first one would be a self created market index out of the dataset used. Therefore a value weighted portfolio under monthly re-balancing is built, denoted as 'own market index'. This index serves as a representation to measure the relative performance between each other, given the subsetted investment universe. However, an U.S. investor does not have the same limitations in his investment universe as is I set in the survey, thus I rather stick to the second plausible market index, the broadly used reference market index of daily returns from the CRSP database. Corresponding data is directly loaded from K. French's data library [1]. This market index captures a broader picture of the U.S. market and is the usually applied market index within relevant literature (e.g. Frazzini & Pedersen (2014), Schneider et al. (2016), Bali, Hu & Murray (2017), Bali et al. (2016)). As this market index comes closer to real world expectations and is mostly accepted by academics, I always stick to that reference in the empirical analysis, the own market index is only used if it is explicitly mentioned in the context.

[1] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`; accessed April 9^{th}, 2018

Beta. Since the market risk - what is always referred to as 'beta' within this reading - is crucial in this empirical survey, so I put high diligence upon its measurement. Academic literature provides different approaches measuring market beta. According to Bali et al. (2016), the most common used approach estimating beta in empirical finance studies is simply a CAPM regression based on historical data of excess portfolio returns against excess market returns, also denoted as physical beta estimation. Other popular methodologies could be beta corrected for non-synchronous trading effects (cp. Scholes (1977)) or Dimson (1979)'s estimate considering infrequently traded stock's. Nonetheless, as Bali et al. (2016) show, the later ones do not necessarily produce better outputs when it comes to the predictability of the market coefficient. Besides physical beta estimation, there exists also broad literature (e.g. French et al. (1983), Duan & Wei (2009), Chang et al. (2012), Buss & Vilkov (2012)) trying to proxy beta on a risk-neutral basis. However, Bali, Hu & Murray (2017) - who also study beta and risk-neutral moments effects - find that those applications rely on contrary assumptions in the relevant context. For example, in the case of French et al. (1983) and Buss & Vilkov (2012) they see their measure as physical instead of ex-ante, as it is computed on historical correlation data. Duan & Wei (2009) and Chang et al. (2012) use the basic assumption that idiosyncratic ex-ante skewness and kurtosis are zero. Given that this work directly estimates risk-neutral moments from the option implied volatility surface without any further separation to exclude additional assumptions, also this beta estimation approach may be inappropriate. As Bali, Hu & Murray (2017) conclude from their literature review to use physical measure to proxy risk-neutral beta, I share the same opinion and implement a rolling physical beta estimation, backward looking over two years of daily returns. I do use a two year time window as the time-varying beta estimate gets thus less volatile. Following Merton (1980), daily return series are used rather than monthly returns to enlarge the sample frequency, which further enhances the estimation accuracy upon covariances. Similarly to Frazzini & Pedersen (2014), a minimum of 252 trading days (one trading year) is required to estimate a stocks two-year physical beta.

Additionally to the rolling physical proxies for investors' market risk expectations, robustness tests using the beta estimation procedure as introduced by Frazzini & Pedersen (2014) are provided (see Appendix B). Their (unadjusted) rolling risk-neutral beta $\hat{\beta}_i^{TS}$ of a firm i is defined as follows:

$$\hat{\beta}_i^{TS} = \hat{\rho}_{i,m} \frac{\hat{\sigma}_i}{\hat{\sigma}_m}, \tag{18}$$

with $\hat{\sigma}_m$ and $\hat{\sigma}_i$ representing the volatilities of the market m and the firm i respectively. $\hat{\rho}_{i,m}$ denotes the correlation between i and m. Note that there are two things to pay attention to. First, correlation and volatilities are computed on a different basis. Second, while volatilities are computed from one-year historical daily log returns $r_{i,t}$, correlations are estimated over a five year lookback window based on three days overlapping log returns $r_{i,t}^{3d}$:

$$r_{i,t}^{3d} = \sum_{k=0}^{2} ln(1 + r_{i,t+k}). \tag{19}$$

With a minimum of 120 trading days for volatilities and 750 for correlations, Frazzini & Pedersen (2014) argue that $r_{i,t}^{3d}$ are more appropiate at estimating correlations to remove non-synchronous trading effects and use DeSantis & Gerard (1997)'s statement that volatilities move faster than correlations to implement the different lookback periods. Then, by generally setting $w = 0.6$ and $\hat{\beta}^{XS} = 1$, Frazzini & Pedersen (2014)'s estimated ex-ante rolling beta is proxied by

$$\hat{\beta}_i = w\hat{\beta}_i^{TS} + (1 - w)\hat{\beta}^{XS}, \tag{20}$$

which should help to reduce the inappropriate influence of outliers (cp. Vasicek (1973)). Anyway, as the 2-year roling historical beta turns out in this survey to predict future realized beta more precise than the Frazzini & Pedersen (2014) beta does, taken exante beta is simply the 2-year physically measured one. Results found using Frazzini

& Pedersen (2014) support the general robustness of the survey and are displayed in Appendix B.

4 Empirical Testing

Within this chapter, the empirical results encountered when testing for ex-ante skewness and kurtosis effects in the given stock market are explained. After having described the used data as well as the methodology behind the ex-ante measures, the empirical part starts by testing for existence of skewness and kurtosis for all stocks in the defined universe, such it is verified that higher distributional moments are not a random appearance but rather a common feature of stock return series. Once it is shown that stock returns are indeed not perfectly normally distributed, I examine and verify the significant existence of the beta anomaly as stated in Frazzini & Pedersen (2014). Later on, when both higher moments as well as beta anomalies are proofed to exist, the two effects are isolated from each other. Doing so, double and triple sorted portfolios are built among clusters of rolling beta with risk-neutral skewness, kurtosis and tail risk, where I simply refer to tail risk as a combination of skewness and kurtosis. Please note, when speaking of beta, always the systematic market risk exposure is meant. The return series from the monthly rebalanced, double and triple sorted portfolios built on rolling measures are then tested unconditionally under CAPM, Fama & French (1993) ('FF3') and Fama & French (2015) ('FF5') asset pricing models. If not highlighted differently, presented alpha values are risk-free excess returns denoted in monthly percent, defined as the regression intercepts. Generally, when displayed in tables, significant alpha values (p-value below 5%) are shown in bold.

From conditional single sort analysis I find support for the existence of the beta anomaly, e.g. under CAPM, the lowest beta portfolio produces a slightly positive alpha at 0.05% monthly, while with increasing beta, alpha turns more and more negative down to -1.14% in the highest beta portfolio, hence I can confirm the findings of Frazzini & Pedersen (2014). I further observe superior significance levels of high beta portfolios, while low-beta portfolio's alpha evolve to be less significant. This finding is supported by the empirical work of Baker et al. (2011). High beta stocks face restrictions and are more costly when it comes to short selling, thus significant under-performance of high beta stocks can be an persistent occurrence of stock markets. Further, when taking a look at the mean excess returns, high beta portfolios under perform the low and mid beta portfolios, which is an very controversially result to basic finance theory, but still a widely recognized phenomena in academic literature (cp. Baker et al. (2011)).

Equally to Schneider et al. (2016), I find that ex-ante skewness, in this case measured non-parametrically, embeds predictive power on both, future realized returns and skewness (see Table 7). By double sorting, I find throughout significant alpha values where the low ex-ante skewness (kurtosis) portfolios can be distinguished from the high skewness (kurtosis) ones. It turns out that stocks with lower ex-ante skewness generate severe risk adjusted under-performance unlike their high (or neutral) counterparts, the mean difference in alpha is 0.53 percentage points monthly (under CAPM). Using the non-parametric ex-ante kurtosis, I find low kurtic firms to produce higher intercepts than the high kurtic opposites, here the average difference is 0.92 percentage points per month. As both measures point out that higher credit risk reflects worse performance, I triple sort on four beta categories and each once on low skewness plus high kurtosis (= high tail risk) and high skewness plus low kurtosis (=small tail risk). From triple sorting, the tail risk anomaly is even stronger pronounced with an average difference in alpha of 1.04 percentage points monthly. The tail risk anomaly is throughout persistent, even under multi factor models as FF3 and FF5. For additional robustness checks, the same examination using Frazzini & Pedersen (2014)'s beta measure is set up, resulting in the same conclusion (see Appendix B). As Frazzini & Pedersen (2014)'s beta is also based on historical return variances and correlations, thus not ex-ante per-se, tests were also made using implied volatility as real risk-neutral variance proxy (cp. Schneider et al. (2016)), results found here also indicate beta and tail risk anomalies.

To support the results under conditional sorts, I make use of a random portfolio technique which I refer to as bootstrapping. The idea behind this method is to randomly choose a set of stocks from the dataset to form a portfolio and then compute the portfolio return series. This procedure is repeated many times to generate a sufficient number of different portfolios, in this survey, usually 1000 random sampled portfolios are generated. With this method the great advantage comes that it is completely free of any assumption concerning characteristics to form portfolios upon, thus random portfolios are per definition free from systematic portfolio forming biases which may occur under conditional portfolio sorts. Once a large number of random portfolios is built, the next step in the bootstrapping is to run a portfolio performance analysis. In this case, it means a simple unconditional regression of excess portfolio returns under CAPM, FF3 and FF5. In difference to the conditional sorting technique, where only a small number of portfolios can be generated to guarantee well established diversification (e.g 10 portfolio when conditionally sorting on rolling betas), random portfolio bootstrapping gives a way greater number of outputs to analyze, which makes the result more robust. However, possible drawbacks of this technique are that it is less realistic to real world applications (who chooses portfolio allocation randomly?), it requires more computational power and may different methods need to be applied in order to isolate the desired effect. Testing random sampled portfolios is nothing new, Fama & French (1993) already used this kind of method to support evidence for their HML and SMB factors. Other examples making use of the random sampling method are Russell et al. (1994) or Ecker (2012). By randomly forming the portfolios, the results are not affected by data snooping bias as stated in Lo & MacKinlay (1990), nor do they inflate data fitting (cp. Lewellen et al. (2010)).

4.1 Initial Tests

At the beginning of the empirical survey I start with some basic tests providing the reader with information about how the declared investment universe performs relative to the CRSP market portfolio, examine if the stock data covers the beta anomaly and test whether the normality assumption in return series is violated. All those introductory tests build the basis for the following, more detailed evaluation of 3rd and 4th moment effects. For regression analysis, always portfolio excess returns are used where the portfolios' log returns are subtracted by the risk-free rate defined as the 1-month U.S. treasury bill rate. The used data for the 1-month T-bill is derived from K. French's data library[2].

First, I start with basic regression analysis on daily returns of a portfolio formed out of all 3651 stocks available within the dataset against the CRSP market index. The portfolio is built by monthly re-balancing, where every stock is weighted according to its current market capitalization. The regression results under CAPM, FF3 and FF5 are shown in Table 1.

Table (1) Regression of dataset portfolio against asset pricing models

	CAPM	FF-3	FF-5
(Intercept)	-0.42^{***}	-0.42^{***}	-0.42^{***}
Mkt.RF	1.02^{***}	1.02^{***}	1.02^{***}
SMB		-0.63^{***}	-0.54^{***}
HML		-0.21^{*}	0.08
RMW			-0.01
CMA			-1.47^{***}
R^2	0.99	0.99	0.99
Adj. R^2	0.99	0.99	0.99

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Interestingly, with the corresponding dataset covering the U.S. stock market re-

[2] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`; accessed April 9^{th}, 2018

stricted to option availability, market capitalization greater 300 Mio. USD and correcting for survivorship bias, the investment-universe portfolio significantly under-performs the CRSP market index within all three asset pricing models. This leads to the suggestion, that also at further testings, most of the intercept results will turn negative. Nonetheless the dataset-portfolio significantly under-performs the index by CRSP, I still stick to the CRSP market portfolio as the preferred regression benchmark since it better fulfills the definitions of the theoretical models, covering broader investment possibilities. Also relevant literature to address the beta and tail risk anomalies (e.g. Frazzini & Pedersen (2014) and Schneider et al. (2016)) use the CRSP market benchmark. Thus, to make the empirical evaluation comparable to this literature, I find that the CRSP is most appropriate in the case of this survey.

4.1.1 Beta Anomaly

Given the study focuses on splitting up asset pricing's beta anomaly into tail risk clusters, it is obligatory to first show empirical evidence for the existence of such an anomaly. Literature states that low beta stocks generate significantly positive alphas while the opposite is true for high beta stocks (e.g. Lewellen & Nagel (2006), Frazzini & Pedersen (2014)). Additionally to those studies, I also put this anomaly testing framework following Schneider et al. (2016) replacing the systematic risk measure beta with the stock option's at-the-money implied volatility. According to Buss & Vilkov (2012), option implied volatility contains forward looking information for systematic risk exposure based on investor's expectations, it may delivers a better picture of asset pricing rather than the regression derived beta. Results on beta anomaly tests using Frazzini & Pedersen (2014) beta instead of the 2-year rolling beta which I usually refer to are attached in Appendix B.

But first I start with historical return analysis and regression frameworks to show that the beta anomaly indeed exists. Therefore I decided to make use of the bootstraping methodology explained earlier, which's procedure is separated into four steps. First, out of the 3651 available stocks, between 300 to 1500 stocks were randomly chosen and value weighted depending on their market capitalization to form a portfolio. In the second step, the performance of the portfolio over the entire dataset time horizon (January 2005 to March 2018) is computed. Step three evaluates the regression results from the portfolio performance within CAPM and FF5 and saves the results into a dataframe. Those three steps were then repeated for 1000 times, which is declared as the fourth and last step in the procedure. By randomly choosing portfolios and repeating the procedure many times, it is designed to delete any known and unknown biases in the portfolio forming process (cp. Lo & MacKinlay (1990), Lewellen et al. (2010)) to get many datapoints of portfolio alphas combined with their betas. The bootstrapping results using CRSP market index as reference under CAPM and FF5 framework are displayed in Fig. 2 and 3 respectively.

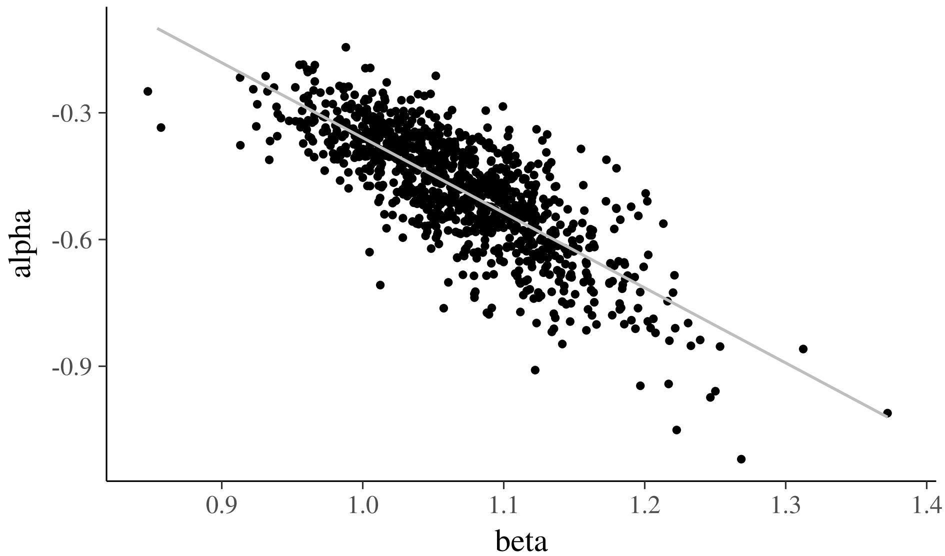

Figure (2) Random sampled portfolios support existence of the beta anomaly. Tested under CAPM, the higher beta the smaller realized alphas.

Table (2) Bootstrapping results from 1000 random sample portfolios tested under CAPM

	N	Mean	St. Dev.	Min	Max
alpha	1,000	−0.48	14.16	−1.12	−0.14
p-value α	1,000	0.001	0.00	0.00	0.18
beta	1,000	1.07	6.21	0.85	1.37
p-value β	1,000	0.00	0.00	0	0
R.squared	1,000	0.94	2.83	0.78	0.98

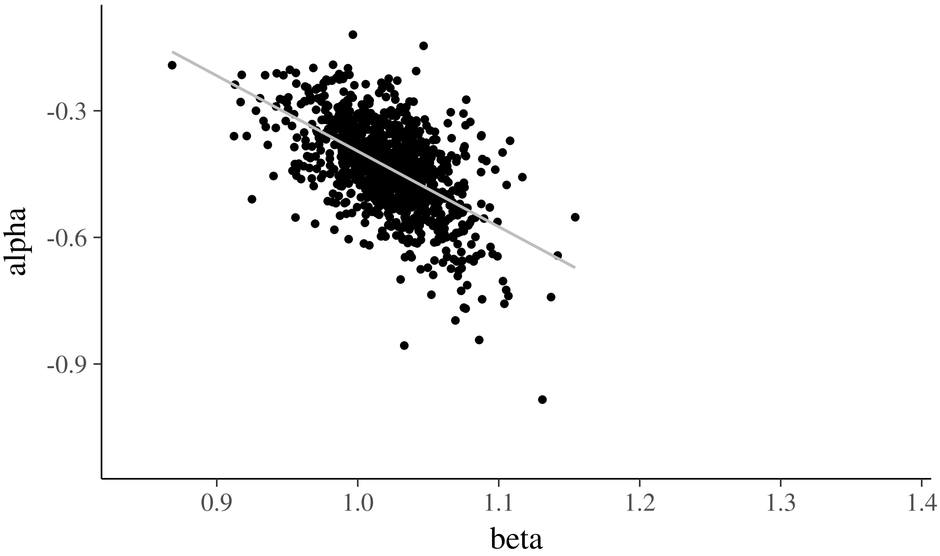

Figure (3) Also under FF5, the beta anomaly clearly exists

Table (3) Bootstrapping results from 1000 random sample portfolios tested under FF5

	N	Mean	St. Dev.	Min	Max
alpha	1,000	−0.43	10.73	−0.98	−0.12
p-value α	1,000	0.002	0.00	0.00	0.22
beta	1,000	1.02	3.37	0.87	1.15
p-value β	1,000	0.00	0.00	0	0
R.squared	1,000	0.96	2.94	0.82	0.99

From the two plots it can be seen, that within both models, portfolios with higher risk exposure tend to under-perform their low beta counterparts. In addition, the figures include a linear fitted line to indicate the direction of the alpha-beta relation. However, when comapring Fig. 2 with Fig. 3 I also observe that alpha values tend to be slightly smaller under Fama & French (2015)'s five factor and more 'clouded' aka less correlated. But still, the beta anomaly exists in CAPM and FF5 settings. Hence, results here support the hypothesis that stock returns are anomalous among systematic risk exposure. To highlight the effect of embedding additional factors to CAPM, I set the axis scale in Fig. 2 equal to Fig. 3. What is clearly observable from visual inspection is, that the coefficients from the mutlifactor model take on explanation potential of the beta coefficients, such that the beta values in FF5 are centered closer around 1 with smaller standard deviation (6.21% vs. 3.37%) than from CAPM testings. Therefore I believe the beta anomaly to be less present in the five factor model than at CAPM. In numbers, the correlation between alpha and beta under FF5 is -0.62 whileas stronger at CAPM with -0.80, both correlations are highly significant with p-values from pearson's paired sample correlation test below 0.1%.

As the first initial test indicated under-performance of the dataset compared to the CRSP market index, none of the bootstrapping portfolios could outperform after risk adjustment. Still, the lower the beta, the greater will be the produced alpha. Information that these results are valid are provided in the bootstrapping summary statistics in Table 2 and 3 where it can be found, that for most portfolios the regression coefficients were highly significant. Only for 4 out of the 1000 regressions CAPM's intercepts produced p-values above the 5% mark (99.6% of alphas were significant), FF5 produced with 7 unsignificant intercepts slightly more (99.3% of alphas were significant). Besides, I find that already CAPM provides a really good fit for the stock universe with an average R^2 of 94.4%, where adding the four supplementary factors from FF5 enhances the mean R^2 only up to 96.1%. Within both models, the linear regression showed highly significant, downward sloping relation between alpha and beta, hence the beta anomaly hypothesis holds. The reader may be interested in how the results look like if the CRSP market benchmark is substituted by the value weighted portfolio out of the given investment universe. Corresponding outputs are captured in Appendix A Fig. 24 and Table 23. Summarized, when using the own market index I clearly see an out-performance of low beta stocks against the benchmark and an under-performance of high beta stocks, the regression coefficients are less significant but the alpha-beta regression still pronounces highly significant existence of the beta anomaly.

Drawback of the used bootstraping technique is that it is built unconditionally, meaning that beta fluctuation over time cannot be captured in this setting. Hence a second, conditional approach is used addressing the risk anomaly. Within this second scheme portfolios are built based on investor expectations approximated by rolling betas, as this comes closer to real world applications. Consequently, I compute for each month and every stock in the dataset the two-year rolling beta under CAPM which is used as the proxy for investor's expectations among systematic risk. Given every stocks' beta estimation over time, portfolios were built conditionally on a value weighted and monthly basis. Different to the bootstraping procedure, portfolios were not built out of random stock picks but now by accumulating together stocks with similar betas. To do so, before portfolios were rebalanced each month, all stocks were ranked among their current beta into ten categories. Ranking 1 represents the stocks with lowest beta up to ranking 10 containing highest beta stocks, where every category contains an equal number of stocks. Based on that clustering, ten portfolios are formed representing low to high beta (P1 to P10). This conditional portfolio forming systematic is used in the following reading under the designation 'single-sort' - stocks are sorted upon on single indicator. Now that portfolios are built in advance upon investor's expectations, this approach can be considered as forward looking. Beta single-sorted portfolios were analyzed in the three relevant asset pricing models - CAPM, FF3 and FF5. Fig. 4 shows how the single-sorted portfolios performed.

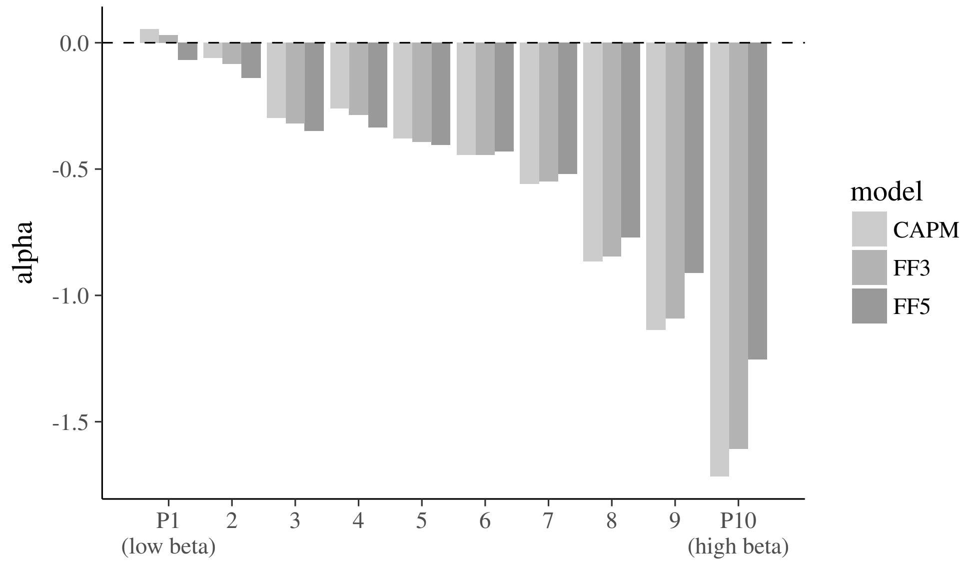

Figure (4) Beta sorted portfolios

Table (4) Conditionally single-sorted portfolios on rolling ex-ante beta

	P1 (low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high beta)
excess return	0.51	0.55	0.4	0.51	0.45	0.44	0.42	0.17	0.05	-0.34
CAPM alpha	0.05	-0.06	**-0.3**	**-0.26**	**-0.38**	**-0.45**	**-0.56**	**-0.87**	**-1.14**	**-1.72**
	-0.35	(-0.54)	(-2.67)	(-2.22)	(-3.41)	(-3.62)	(-3.64)	(-4.58)	(-5)	(-4.75)
FF3 alpha	0.03	-0.08	**-0.32**	**-0.29**	**-0.39**	**-0.44**	**-0.55**	**-0.85**	**-1.09**	**-1.61**
	-0.2	(-0.81)	(-3.06)	(-2.65)	(-3.64)	(-3.66)	(-3.65)	(-4.66)	(-5.16)	(-5.11)
FF5 alpha	-0.07	-0.14	**-0.35**	**-0.33**	**-0.41**	**-0.43**	**-0.52**	**-0.77**	**-0.91**	**-1.25**
	(-0.48)	(-1.36)	(-3.38)	(-3.12)	(-3.76)	(-3.56)	(-3.46)	(-4.38)	(-4.72)	(-4.5)
beta (ex-ante)	0.54	0.76	0.88	0.99	1.09	1.19	1.3	1.42	1.59	1.93
beta (realized)	0.58	0.79	0.91	0.99	1.07	1.15	1.27	1.34	1.53	1.78
volatility	13.02	15.69	17.84	19.51	20.94	22.44	24.92	26.72	30.67	37.18
skewness	-0.17	-0.27	-0.24	-0.27	-0.41	-0.44	-0.38	-0.53	-0.65	-0.4
kurtosis	14.09	10.37	13.71	14.33	10.32	7.78	8.35	7.14	11.12	11.07
PTR	0.47	1.31	2.01	2.26	2.79	2.65	2.62	2.48	1.85	0.84

Observable from this examination is that likewise bootstrapping, within the forward-looking built portfolios using rolling betas, alphas seem to decline with increasing beta. The regression results presented are mainly significant, detailed information on the regression outputs are displayed in Table 4. The results here are in line with the broad consensus of empirical finance literature(e.g. Lewellen & Nagel (2006), Baker et al. (2011), Frazzini & Pedersen (2014), Schneider et al. (2016)). The beta anomaly cannot be removed by setting the testing framework conditionally, the risk adjusted performance will still be inversely related to the exposure of systematic market risk. This relation holds even when testing under FF3 and FF5, where the anomaly could be reduced only portion wise. When putting an eye on absolute performance, I share the same opinion as Baker et al. (2011) that the empirical security market line is flat for low to mid beta stocks or even downward sloping for the high beta side. I further find favors for the arguments of short selling restrictions such that risk adjusted (under) performance finds strong significance the higher the beta, whereas low beta side, allowing investors to bet on positive alphas without taking on short positions, is less significant and less anomalous as it is easier to realize (cp. Baker et al. (2011)). This is also seen as a source for the downward sloping security market line.

Table 4 implicates a robustness check for the ex-ante beta measure. The 2-year rolling beta from CAPM regression is by definition not ex-ante, however, this beta estimating method is widely used (cp. Bali et al. (2016)). Hence I use it as the proxy for 'ex-ante' beta. In Table 4 it can be seen that this estimation evolves to perform very well. Within each portfolio, the mean ex-ante beta comes very close to the portfolios realized beta, thus I can confirm the predictive ability of the ex-ante beta measure used. Interestingly, this beta approximation turns out to predict realized beta more precisely than the more extensive constructed one by Frazzini & Pedersen (2014), which is also the reason why the physical one is the preferred ex-ante beta. Given that the ex-ante beta correctly predicts the risk categories, meaning that realized beta is monotonically increasing from P1 to P10 without any exceptions, it is not surprising that Baker et al. (2011) conclude that there is no difference in the beta anomaly whether one tests under conditional nor unconditional settings, correspondingly, the conclusion from unconditional bootstrapping equals conditional single-sorting.

In Table 4, PTR stands for the portfolio turnover rate. PTR can be interpreted as the percentage of how much from the portfolio's volumina was traded during the year relative to the total portfolio value. This measure is computed differently dependent on which authority to follow, the PTR estimated here is based on the definition from the United States Security Exchange Commision (SEC)[3]:

$$PTR = \frac{min(vol_{in}, vol_{out})}{PF_{value}}, \tag{21}$$

with vol_{min} as the total amount of securities purchased during the year, vol_{out} the sales respectively and PF_{value} the average portfolio value over the given year. Realized turnover rates in the beta sorted portfolios are throughout high, only the very outer ones show a PTR below one. This is caused by separating the stock universe into small clusters, thus stocks often switch between two neighbored portfolios form month to month (e.g. from P5 to P6, and in the next month back to P5). This behavior causes unnecessarily high turnover rates for real world practitioners, hence for smart beta strategies I would personally recommend less beta clusters to categorize portfolios to keep PTR and corresponding transaction costs low. Anyway, in an academic perspective the finer separation makes throughout sense as the beta anomaly can be analyzed in more detail and with more robustness.

In the same way as Schneider et al. (2016) reconstruct the beta anomaly into a volatility anomaly, I use option implied at-the-money volatility as the stock's traded measure for systematic risk exposure. The advantage of using option implied volatility comes from the options direct dependence among future price expectations. Thus instead of creating risk expectations out of historical return series, options allow to measure priced-in and naturally forward looking investor expectations at every point in time. Nevertheless, in hand with this feature comes the drawback, that every stock analyzed requires frequently traded options available, introducing research restrictions

[3] `https://www.nasdaq.com/investing/glossary/p/portfolio-turnover-rate`; accessed June 26^{th}, 2018

as not every traded stock has the feature of option availabilty. Following the same procedure as at single-sorting stocks on beta, portfolios on opition implied volatility were built. Fig. 5 tells the same story as Fig. 4, the greater the investor's expectations on returns, the smaller will be the realized ones. The risk anomaly is consistent, no matter if built on beta or on ex-ante volatility. Likewise, the realized empirical security market line seems flat to downward sloping, as shown in Table 5. Still under FF5, the low volatility portfolio (P1) produces 2.2% of higher risk adjusted return than its high risk opposite (P10). Also in absolute terms, P1 generates with a mean excess return of 0.50% a better performance than P10 facing -1.46% monthly. The robustness for the hypothesis of short selling restrictions making high beta anomaly significant and low beta anomaly insignificant is confirmed by the ex-ante volatility, note that significant alphas (p-value $< 5\%$) in Table 5 are displayed in bold. What can also be taken from Table 5 is the capability of (total) ex-ante volatility predicting future realized volatility, while the ex-ante volatility does not fully correctly predict the exact value, however it is definitely a good predictor for the volatility category, such that the realized volatility is monotonically increasing from P1 to P10 without any mis-clustering. Decomposing ex-ante volatility into systematic and idiosyncratic parts may further push precession in predictability.

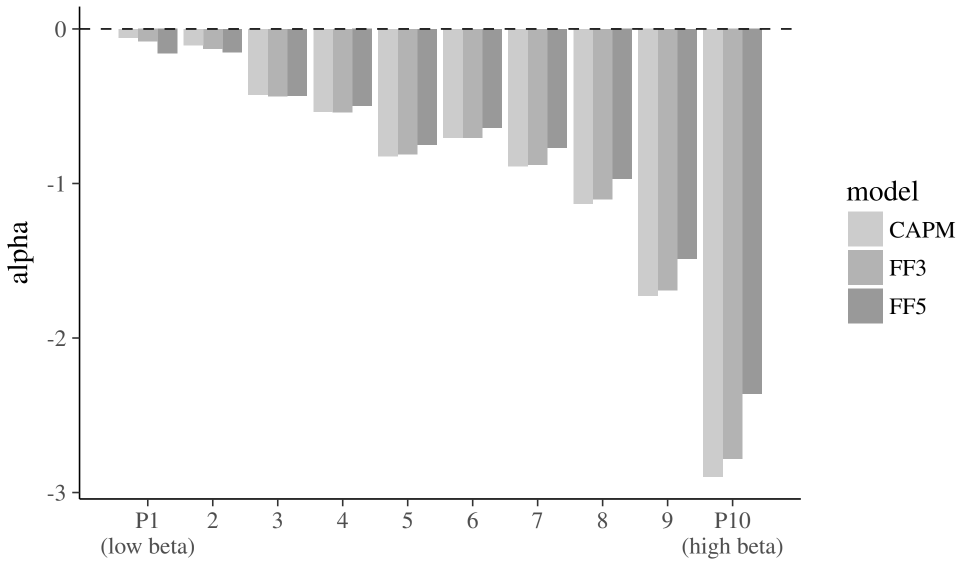

Figure (5) Implied volatility sorted portfolios

Table (5) Portfolios conditionally sorted on ATM implied volatility

	P1 (low i.vol)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high i.vol)
excess return	0.5	0.63	0.41	0.35	0.13	0.31	0.2	0.02	-0.45	-1.46
CAPM alpha	-0.06	-0.11	**-0.43**	**-0.54**	**-0.82**	**-0.7**	**-0.89**	**-1.13**	**-1.73**	**-2.9**
	(-0.5)	(-1.1)	(-4)	(-4.16)	(-4.85)	(-3.76)	(-3.78)	(-4.12)	(-5.02)	(-5.9)
FF3 alpha	-0.08	-0.13	**-0.43**	**-0.54**	**-0.81**	**-0.7**	**-0.88**	**-1.1**	**-1.69**	**-2.78**
	(-0.82)	(-1.42)	(-4.12)	(-4.23)	(-4.95)	(-3.89)	(-3.86)	(-4.22)	(-5.19)	(-6.24)
FF5 alpha	-0.16	-0.15	**-0.43**	**-0.5**	**-0.75**	**-0.64**	**-0.77**	**-0.97**	**-1.49**	**-2.36**
	(-1.81)	(-1.72)	(-4.1)	(-3.94)	(-4.7)	(-3.64)	(-3.53)	(-3.92)	(-4.85)	(-5.8)
volatility (ex-ante)	19.46	25.23	28.88	32.16	35.35	38.81	42.75	47.57	54.9	74.54
beta (realized)	0.7	0.93	1.06	1.13	1.21	1.28	1.38	1.45	1.62	1.82
volatility (realized)	14.25	18.23	20.61	22.15	24.05	25.57	28.04	30.09	34.12	40.64
skewness (realized)	-0.08	-0.24	-0.31	-0.44	-0.53	-0.48	-0.51	-0.46	-0.66	-0.55
kurtosis (realized)	14.65	10.86	8.86	8.91	7.57	8.34	8.54	7.6	10.43	11.68

significant alpha values (p-value $<$ 5%) are displayed in bold

To put it in a nutshell, this section delivers empirical evidence for the asset pricing risk anomaly. High beta stocks under-perform significantly their low beta counterparts, both in absolute and risk adjusted terms. Low risk portfolios may produce slight positive alphas (e.g. P1 0.05% monthly sorted on beta, tested under CAPM) but the market fails to compensate additional risk taken. The anomaly is stronger pronounced on the high risk side than on the lower side and remains consistent under CAPM, FF3 and FF5.

4.1.2 Empirical Skewness and Kurtosis

Already in Section 2, the theoretical effects of non-normality measured as ex-ante skewness and kurtosis upon asset prices are discussed. Here I bring first initial tests which asset pricing structures arise when putting an eye on both of them.

The very first related test is to examine significant existence of skewness and kurtosis within the return series. From the simple one sample t-tests below (Table. 6), realized skewness of returns turns out to be significant negative whereas excess-kurtosis is significant positive. Consequently, CAPM's, FF3's and FF5's basic assumption of normally distributed stock returns does not hold.

Table (6) T-test upon realized skewness and kurtosis of stock returns

	skewness	kurtosis
mean	-0.17^{***}	24.62^{***}
	(-4.67)	(24.00)
df	3646	3646
p-value	0.00	0.00
H1	<0	>0

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Now as it is shown that skewness and kurtosis naturally exist in stock returns of the dataset, I want to know how the two measures influence asset pricing. Recapping from Section 2, negative skewness and leptokurtosis are directly linked to higher tail risk loadings, hence investors should require adequate risk compensation in order to take on those positions, otherwise portfolios will generate systematic under-performance with increasing tail risk. To examine the real world situation, same single-sorting testing frameworks as for the risk anomly in section 4.1.1 were applied. Fig. 6 gives insights of the effect whether investors trade on certain skewness or kurtosis levels, the moments are measured on a rolling basis using once (i) one-year historical returns and second (ii) from current option implied volatility surface taking non-parametric approximation. Table 7 and 8 give a summary of the regression results from ex-ante sorted portfolios.

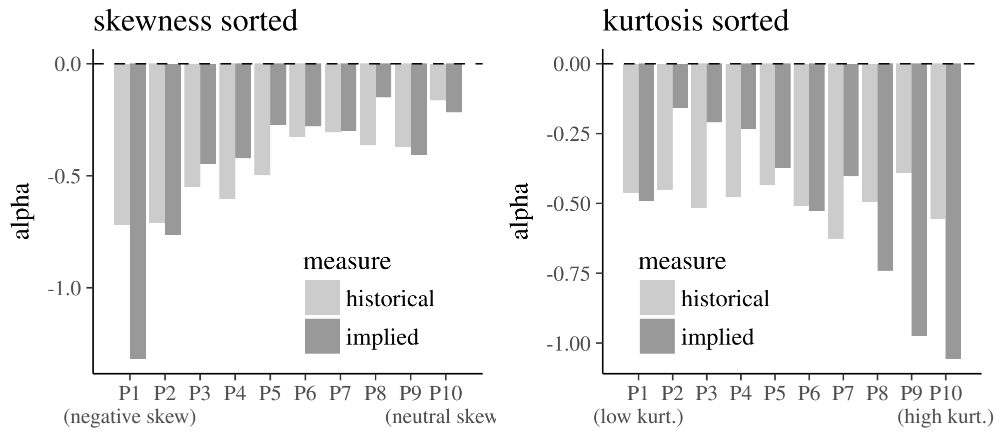

Figure (6) Alphas realized sorting on skewness / kurtosis

Table (7) Regression summary of ex-ante skewness sorted portfolios

	P1 (low skew)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high skew)
excess return	0.08	0.18	0.38	0.25	0.32	0.46	0.45	0.36	0.34	0.51
FF5 alpha	**-0.72**	**-0.71**	**-0.55**	**-0.6**	**-0.5**	**-0.33**	**-0.31**	**-0.36**	**-0.37**	-0.16
	(-4.03)	(-4.76)	(-3.58)	(-4.56)	(-4.66)	(-2.93)	(-2.68)	(-2.51)	(-2.2)	(-0.87)
skewness (ex-ante)	-1.19	-0.64	-0.54	-0.48	-0.43	-0.39	-0.34	-0.3	-0.25	-0.17
skewness (realized)	-0.64	-0.57	-0.57	-0.54	-0.43	-0.27	-0.38	-0.58	-0.29	-0.57
beta (realized)	1.04	1.08	1.11	1.07	1.02	0.99	1	1	0.97	0.98

significant alpha values (p-value $<$ 5%) are displayed in bold

Table (8) Regression summary of ex-ante kurtosis sorted portfolios

	P1 (low kurt)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high kurt)
excess return	0.23	0.64	0.63	0.56	0.47	0.3	0.45	0.07	-0.22	-0.27
FF5 alpha	**-0.49**	-0.16	-0.21	-0.23	**-0.37**	**-0.53**	**-0.4**	**-0.74**	**-0.98**	-1.06
	(-2.94)	(-1.09)	(-1.72)	(-1.96)	(-3.4)	(-4.09)	(-3.15)	(-4.81)	(-5.04)	(-4.15)
kurtosis (ex-ante)	-0.85	6.35	9.03	10.9	12.85	15.02	17.69	21.57	28.41	76.83
kurtosis (realized)	9.44	13.8	12.52	8.93	10.1	9.59	8.28	8.63	12.69	17.19
beta (realized)	0.99	0.98	0.97	0.97	1.03	1.08	1.1	1.12	1.1	1.16

significant alpha values (p-value $<$ 5%) are displayed in bold

The portfolios were built each upon lowest skewed (kurtic) stocks indicated with

P1 up to highest skewed (kurtic) stocks captured by P10. Observable in Fig. 6 is that with increasing tail risk - negative skewness and higher kurtosis - portfolio performance worsens. This plot also highlights the power of ex-ante measures. While building on historical skewness and kurtosis the performance cannot be distinguished that well, taking ex-ante proxies allows to clearly separate the impact of low to high tail risk. Therefore, risk-neutral measures should be of high relevance for practitioners. The majority of the regression coefficients derived here (see Table 7 and 8) are mainly significant (p-value $< 5\%$) and highlighted in bold with t-values in brackets.

Campbell et al. (2008) show that stocks facing high expectations on credit risk realize lower returns than expected under CAPM and FF3, referring to this empirical phenomena as the distress anomaly or the 'distress puzzle'. They find possible argumentation for the existence of this anomaly in the credit risk aversion among investors. From chapter 2 it is argued that one can directly link ex-ante tail risk to credit risk through the Merton (1974) model. Now, with utility theory it can be shown that investors are averse among this credit risk which coincides the finding of Campbell et al. (2008). Equally to Campbell et al. (2008) I derive that empirically testing uncovers that the market does not compensate this distress aversion, as I see both, under ex-ante skewness and ex-ante kurtosis, higher tail risk loadings lead to lower returns. Among others, further empirical observations of the 'distress puzzle' can be found in Dichev (1998), reporting a negative relation between stock returns and default risk measured by Altmann (1968) Z-Score and Ohlson (1980) O-Score. Nevertheless, the academic discussion about the 'distress puzzle' is broad and comes to mixed results whether it really exists or not. However, the single sorting on ex-ante skewness and kurtosis do give support for the persistence of the 'distress puzzle' such that it is not resolved under those measures. Still, as I found the beta anomaly to be dominant, credit risk loadings may vary over certain portfolios such that by single-sorting on tail risk the desired effect may be ambiguous, this is why later on the double sorting method is implemented.

Supporting the first results from this single-sorting and to derive additional understanding of the dataset, I apply the unconditional bootstrapping technique. Therefore I compute complementary to the portfolio regression outputs the realized skewness and kurtosis. To distinguish high skewed (kurtic) firms from low skewed (kurtic) firms, the portfolios are clustered into quartiles. Using again 1000 portfolio forming repetitions, I take only the first and fourth skewness (kurtosis) portfolios, thus the number of analyzed portfolios is reduced to 500. Fig. 7 displays the corresponding outcome plus additionally fitted lines for the two categories each.

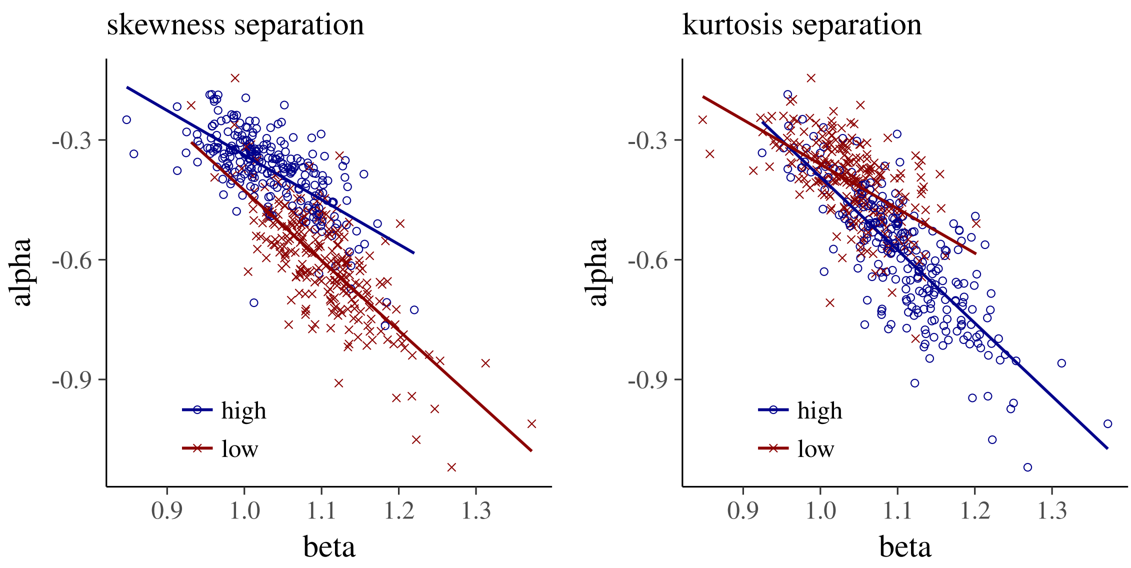

Figure (7) Separating bootstrapping results into skewness and kurtosis clusters

The plots in Fig. 7 show that separating solely into skewness and kurtosis does not give a that clear picture to make a valid statement about whether one category performs significantly worse than the other. There are only slight tendencies visible when putting an eye on the linear fits. In both plots, the fitted line representing higher tail risk seems to fall below its counterpart meaning an indication for skewness and kurtosis effects on realized returns. Interestingly, the linear fits of both data clouds in one plot indicate different slopes. Respectively, each data cloud facing higher tail risk realizes a steeper regression line compared to the low tail risk opposite. At this point the reader may compare Fig. 7 here with the plots Fig. 4 and 5, just from visual inspection I think that the asset pricing lines among beta are rather a downward sloping curve than a linear model, which would be also supported by the plots about risk anomalies but also by the different slopes in the bootstrapping plots.

At least, this statement is not unknown and already enjoys discussion in finance research (cp. Baker et al. (2011)). This downward sloping beta anomaly is also found in several other sections within this work. Additionally, first incentives are given that higher credit risk induces worse performance for similar beta levels, as for both plots the linear fitted line of the higher tail risk portfolios falls underneath the low tail risk portfolios. But since these plots do not correct for the beta anomaly, I cannot already make a clear statement given that the tail risk effects are not isolated from the portfolio's return dependence on beta.

Nonetheless visual inspection is not all, I introduce a testing framework upon this separation of the bootstrapping results. Consequently, finding systematic risk as a main driver of under-performance, realized skewness/kurtosis-only effects on asset prices are tried to be isolated. As the linear regression in Fig. 2 and 3 seems to fit quite well in that model, I apply for this setting the simplified assumption of linear dependence between alpha and beta. In the next step, alpha values are normalized by beta, such that the beta dependent influence is removed and all alpha values are scaled to an beta of 1. By simply dividing the portfolios' *i* alpha values by their respective beta, they are all linearly scaled to the same beta where the systematic volatility effect is removed, isolating skewness and kurtosis effects.

$$\alpha_i^{norm} = \frac{alpha_i}{beta_i} \tag{22}$$

In Fig. 8 the bootstrap data were categorized into low versus high skewness, kurtosis and tail risk, where the first two named are formed from the outer 25% and the latter one as a combination from dividing skewness and kurtosis into half. Following, two sample t-tests were run to examine whether normalized alphas from portfolios with higher realized non-normality measure (α^{norm}_{high}) significantly under(over)-perform their counterparts (α^{norm}_{low}), see Table. 9.

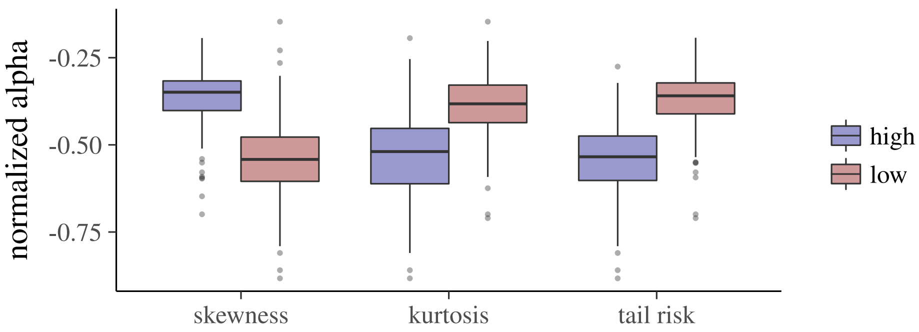

Figure (8) Monthly normalized alphas analyzed when categorized non-normality measure

Under skewness classification, neutral (=high) skewed firms are expected to distinctively perform better than the very negative (=low) skewed ones. Therefore, the t-test hypothesis for α^{norm} categorized by skewness are defined as

$$\begin{aligned} H_0 &: \mu(\alpha^{norm}_{low}) \geq \mu(\alpha^{norm}_{high}) \\ H_1 &: \mu(\alpha^{norm}_{low}) < \mu(\alpha^{norm}_{high}). \end{aligned} \tag{23}$$

For kurtosis and tail risk clustering, the tested hypothesis areth other way around with

$$\begin{aligned} H_0 &: \mu(\alpha^{norm}_{low}) \leq \mu(\alpha^{norm}_{high}) \\ H_1 &: \mu(\alpha^{norm}_{low}) > \mu(\alpha^{norm}_{high}), \end{aligned} \tag{24}$$

the corresponding t-test outputs are displayed in Table 9 below:

Table (9) Two sample t-test (one-sided) for normalized alphas of different categories

	$\mu(\alpha_{low}^{norm})$	$\mu(\alpha_{high}^{norm})$	t	p-value
skewness	-0.53***	-0.36***	-21.74	0.00
kurtosis	-0.40***	-0.50***	-15.47	0.00
tail risk	-0.38***	-0.55***	-24.584	0.00

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

The suspicion derived from Fig. 8 that normalized alphas can indeed be separated among non-normality risk levels is confirmed by the t-test in Table 9. By normalizing for the beta anomaly, significant evidence evolves that the portfolios with higher non-normality risk face lower returns. However, the beta effect upon asset prices will still be stronger whereas the non-normality effect here is rather small. The mean of the normalized alphas from the t-test can be interpreted as the average risk adjusted preformance difference between portfolios with high and low non-normality risk. To put it into an economically useful perspective, these risk-free returns are displayed in monthly interests. Concretely, the annual risk-less out-performance when betting against non-normality risk over the time horizon of January 2005 to March 2018 would have been on average 1.90% for neutral-skewed portfolios, 1.47% for low-kurtic portfolios and 2.08% at the low tail risk portfolios. Considering that portfolios were never re-balanced during this empirical analysis, these risk-free returns may be economically significant, however still very small. Nonetheless, a building-on-expectations approach will be closer to real world applications, which is exactly what is achieved within the following sections.

Summarizing, in this section simple structures between realized as well as ex-ante skewness (kurtosis) and performance were analyzed. I notice correlation of non-normality risk with realized alpha values, meaning that CAPM, FF3 and even FF5 are not able to capture those risk portions within their models. Generally speaking, the greater the tail risk taken, the worse will be risk adjusted performance of a well diversified portfolio, which is in line with the results of eg. Schneider et al. (2016) or

Bali, Hu & Murray (2017). As market risk exposure is the main driver of returns, it is obligatory to isolate tail risk from the beta dependence, otherwise any conclusions will be biased as high beta under-performance dominates the skewness/kurtosis effect in unconditional regressions. Key takeaway here, skewness and kurtosis naturally exist in the U.S. stock market and have significant impact on asset prices. With this conclusion I set up the next testing framework in Section 4.2 using ex-ante measures to build portfolios by sorting on beta and non-normality.

4.2 Building on Expectations

Section 2 highlighted the theoretical background and thoughts of how non-normalities in return distributions relates to asset pricing anomalies. Section 4.1 then tests for basic structures in U.S. stock return series. Following these results, beta anomalies do exist and violation of the normal assumption is a significant phenomena. Besides, first insights about the tail risk puzzle and its influence on stock prices are derived. What is going to be presented in the following reading is a more detailed and closer to real world practice approach to empirically evaluate the existence and profitability of betting against beta (BaB) strategies versus betting against and tail risk (BaB+TR) to further lift portfolio performance. Generally speaking, the applied methodologies are based on monthly re-balancing, building value weighted portfolios upon different expectations to then evaluate portfolio return series and analyze them under CAPM, FF3 and FF5 regression models. Ex-ante skewness and kurtosis are estimated by the non-parametric approach discussed in Section 3.1. Establishing risk-neutral measures allows forming portfolios based on current investor expectations rather than on past return data, which put this work closer to real world behavior as investors usually allocate their portfolios based on their expectations about future risk-return patterns. That the risk-neutral approach is also more powerful than betting on historical return distributions was already shown in Fig. 6.

4.2.1 Double Sorts

Beta, implied volatility, skewness and kurtosis turn out to show significant impact on portfolio performance, combining normality assuming measures (= beta and implied volatility) with non-normality coefficients (= skewness and kurtosis), in order to differentiate portfolio performances. Similar to section 4.1 I try to isolate the effects of non-normality risk from high beta anomalies. With the double sorting mechanism, systematic market risk proxies are computed monthly and combined with one of the non-normality measures.

Beta-Skewness Sorts. At beta-skewness sorts, the beta sorted portfolios are clustered among ex-ante skewness levels and compared within each beta category (P1 = low beta up to P4 = high beta). Each month, skewness clusters were built by categorizing the stocks' current skewness levels into quartiles. Fig. 9 displays comparison of the top 25% ex-ante skewed firms to the low 25% ones for each category of rolling beta, where every plot shows realized risk adjusted performance tested in the corresponding asset pricing model. As result, portfolios facing superior non-normality produce severe negative alphas in contrast to their counterparts with close to neutral skewness do. Further, the gap between low and high skewed firms seems to disappear among increasing beta, hence betting against skewness looks like to be most profitable at low beta portfolios.

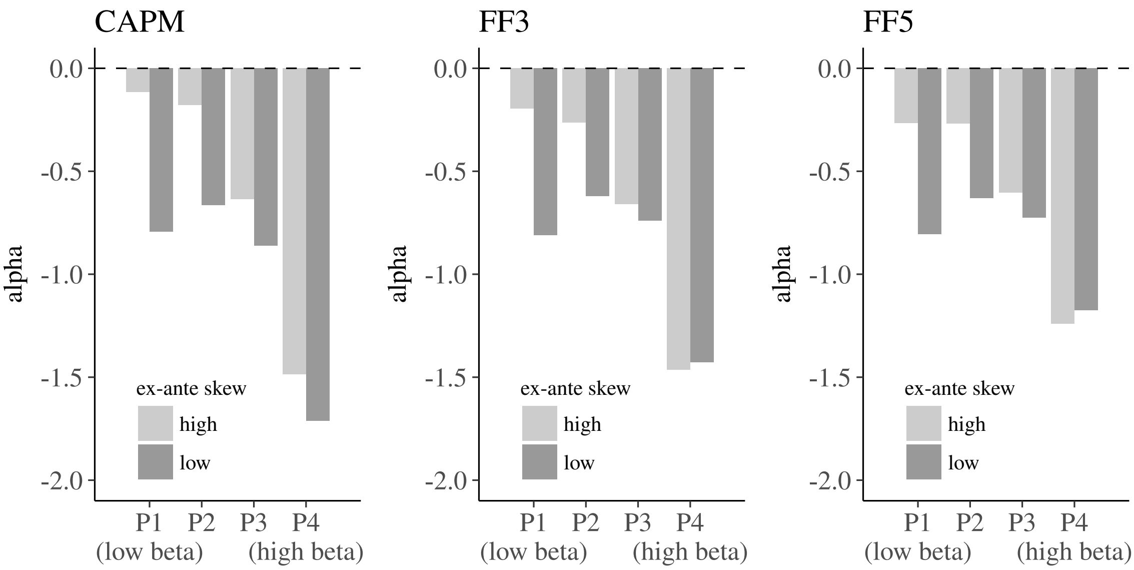

Figure (9) Double sorted portfolios upon beta and skewness

Table (10) Double sorted portfolios on beta and ex-ante skewness

	low ex-ante skew				*high ex-ante skew*			
	P1 L (low beta)	P2 L	P3 L	P4 L (high beta)	P1 H (low beta)	P2 H	P3 H	P4 H (high beta)
excess return	-0.15	0.23	0.18	-0.33	0.42	0.63	0.35	-0.28
CAPM alpha	**-0.79**	**-0.67**	**-0.86**	**-1.71**	-0.1	-0.33	**-0.64**	**-0.81**
	(-2.78)	(-2.96)	(-3.19)	(-4.42)	(-0.72)	(-0.91)	(-2.7)	(-3.91)
FF3 alpha	**-0.81**	**-0.62**	**-0.74**	**-1.43**	-0.19	-0.26	**-0.66**	**-1.47**
	(-2.85)	(-2.88)	(-3.07)	(-4.75)	(-1.39)	(-1.45)	(-2.85)	(-4.01)
FF5 alpha	**-0.8**	**-0.63**	**-0.72**	**-1.18**	**-0.27**	-0.27	**-0.6**	**-1.24**
	(-2.84)	(-2.92)	(-3.01)	(-4.35)	(-1.99)	(-1.47)	(-2.64)	(-3.62)
skew (ex-ante)	-0.87	-0.77	-0.73	-0.76	-0.23	-0.24	-0.23	-0.23
beta (ex-ante)	0.72	1.02	1.28	1.72	0.65	1	1.26	1.64
beta (realized)	0.8	1.11	1.29	1.71	0.67	1	1.22	1.49
volatility	19.71	24.08	28.12	37.62	14.88	21.63	26.22	33.64

significant alpha values (p-value $<$ 5%) are displayed in bold

These results are very interesting in the way that building portfolios on expectations still leads to the skewness effect, where one would initially expect from utility theory that investors should put a premium on negative skewed firms. Anyway, I find that it is exactly the opposite way around, negative skewness leads to higher overall risk loadings causing the portfolio performance to fall steadily below return expectations.

This shows, that with skewness, stocks are exposed to an additional risk source which is not captured by common asset pricing models, where the systematic risk component is overestimated, initiating too high return expectations. I further observe that ex-ante skewness is the dominant effect on the low beta side, whereas on high beta levels, clearly the beta anomaly overweights. By adding more factors, the skewness effect even disappears for high beta stocks under FF3 and FF5. Considering Table 10, representing the analysis output in numbers, the gap between the two skewness categories tested under FF3 in P1 is 0.62% of risk adjusted return monthly, while high beta's gap is solely 0.04%. The results found show high significance within all three tested models, note that significant alphas in Table 10 are highlighted in bold and t-values are in brackets. As the results provided here are derived from portfolios built during the observation horizon with re-balancing according to rolling ex-ante expectations, the reader can see this analysis as an performance back-test of long-only asset management strategies before trading costs.

To summarize, it can be said that common asset pricing models systematically produce biased results on low beta stocks when ex-ante skewness is non-zero. For high beta portfolios I see a way stronger pronunciation of the beta anomaly, but cannot distinguish portfolio performance according to skewness expectations. CAPM, FF3 and FF5 fail to capture skewness causing a throughout overestimation of expected stock returns in both risk adjusted and absolute terms. Key takeaway for long-only investors, avoiding skewness makes particularly sense for low to mid beta portfolios, it can be easily achieved by accounting for stocks' put-call parities and adds additional performance, e.g. 6.84% additional excess return yearly for low beta portfolios. Hence smart beta strategies can lift both, their risk adjusted and absolute performance by betting against skewness.

Beta-Kurtosis Sorts. Since kurtosis is defined as the second component of tail risk, same portfolio building setup as above is constructed. The original anticipation of the kurtosis impact is connected to the outcomes of beta-skewness sorts, alpha values are prospected to decrease with increasing ex-ante kurtosis. Fig. 10 shows that ex-ante kurtosis indeed negatively predicts stock performance.

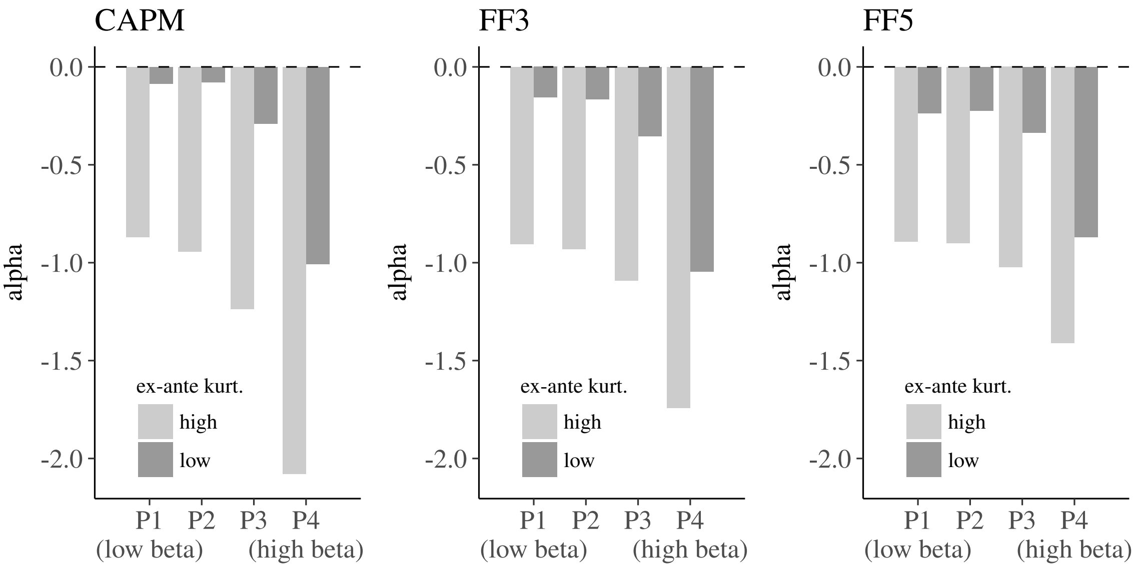

Figure (10) Double sorted portfolios upon beta and kurtosis

Table (11) Summary statistics of beta-kurtosis sorted portfolios

	low ex-ante kurtosis				*high ex-ante kurtosis*			
	P1 L (low beta)	P2 L	P3 L	P4 L (high beta)	P1 H (low beta)	P2 H	P3 H	P4 H (high beta)
excess return	0.46	0.74	0.69	0.18	-0.2	-0.08	-0.18	-0.65
CAPM alpha	-0.08	-0.08	-0.29	**-1.01**	**-0.87**	**-0.94**	**-1.24**	**-2.08**
	(-0.52)	(-0.38)	(-1.19)	(-3.01)	(-3.73)	(-4.77)	(-4.15)	(-4.49)
FF3 alpha	-0.15	-0.17	-0.35	**-1.05**	**-0.9**	**-0.93**	**-1.09**	**-1.74**
	(-1.03)	(-0.84)	(-1.49)	(-3.21)	(-3.92)	(-4.97)	(-4.27)	(-4.87)
FF5 alpha	-0.23	-0.23	-0.34	**-0.87**	**-0.89**	**-0.9**	**-1.02**	**-1.41**
	(-1.62)	(-1.14)	(-1.45)	(-2.8)	(-3.88)	(-4.82)	(-4.03)	(-4.46)
kurtosis (ex-ante)	5.85	2.79	-0.86	-0.64	11.76	12	12.04	12.07
beta (ex-ante)	0.66	1.01	1.26	1.65	0.69	1.01	1.26	1.63
beta (realized)	0.67	1.01	1.21	1.47	0.83	1.07	1.3	1.77
volatility	14.96	22.01	26.27	32.4	19.16	22.92	28.74	40.09

significant alpha values (p-value $< 5\%$) are displayed in bold

Different to beta-skewness sorts, the kurtosis effect also remains for high beta stocks and is only slightly smaller for those portfolios. Table 11 shows the exact regression results. Low ex-ante kurtic portfolios evolve to produce throughout positive excess returns, while all the high ex-ante kurtosis are negative in absolute performance. Significance levels are slightly less but still high when compared to beta-skewness sorts. Notably, similar to the beta-skewness sorts, always the higher tail risk cluster appears to show less significant alphas. When linking parallels to the high beta anomaly, where the existence can be argued to persist due to short selling costs, one could also see a possible reason for the two tail risk effects to remain as the higher risk may be costly or restricted to short sell (cp. Baker et al. (2011)) or follow explanations of Campbell et al. (2008)'s 'distress puzzle'.

The alpha gap between the two kurtosis levels is clearly observable for all three asset pricing models. The addtional factors in the Fama-French models could capture only a slight part of kurtosis related mis-performance, so that the average intercept-gap between low and high kurtosis is 0.92 percentage points monthly at CAPM down to 0.64 in FF5. As consequence, the extra 4 factors in FF5 could reduce the kurtosis anomaly only minimal. This is another evidence that also more sophisticated models are not able to capture non-normality risk. At least it is niceley visualized, of how the size, value, profitability and momentum factors portion wise reduce the intercepts towards zero. In all cases, the mentioned downward sloping shape among beta levels is again clearly visible.

Different to betting against ex-ante skewness, the kurtosis effect is observable throughout all levels of beta. In terms of asset management, long-only investors could achieve on average 64bps higher monthly alpha by simply avoiding high kurtosis stocks within their portfolios. Without transaction costs considered, the smart beta strategy with long only position in low beta and kurtosis would have enabled a yearly excess return of 5.52% compared to -7.8% from the most unfavorable portfolio facing high beta plus high kurtosis, the difference in FF5 alphas between those two portfolios is thus very high with 1,18% monthly.

The double sorting results are mainly in line with the outcomes found by Schneider et al. (2016). But still, from double sorts, neither skewness nor kurtosis were able to completely remove the beta anomaly. However, by accounting for those implied moments, the risk anomaly can be at least partially released. Given strong evidence for non-normality anomalies in asset pricing models, I state the bias to be systematic in common asset pricing models. The double sorts up to here were built based on rolling ex-ante beta estimates computed from two year historical returns, Table 4,10 and 11 show that the estimated ex-ante beta turns out to be very robust and precise in predicting future realized betas. For additional matters of robustness, I use option implied ATM volatility as the per-se ex-ante measure of systematic risk exposure.

Substituting Beta by Implied Volatility. In general, the picture using implied volatility instead of beta looks pretty much the same (cp. Fig. 11). Both tail risk measures indicate that the greater the expectation about the return distribution differs from normal, the worse will be the risk-less performance component. The tail-risk anomaly is still consistent when building on implied volatility, even it is way smaller than before, but again, I observe a stronger pronunciation of the gap for kurtosis than skewness.

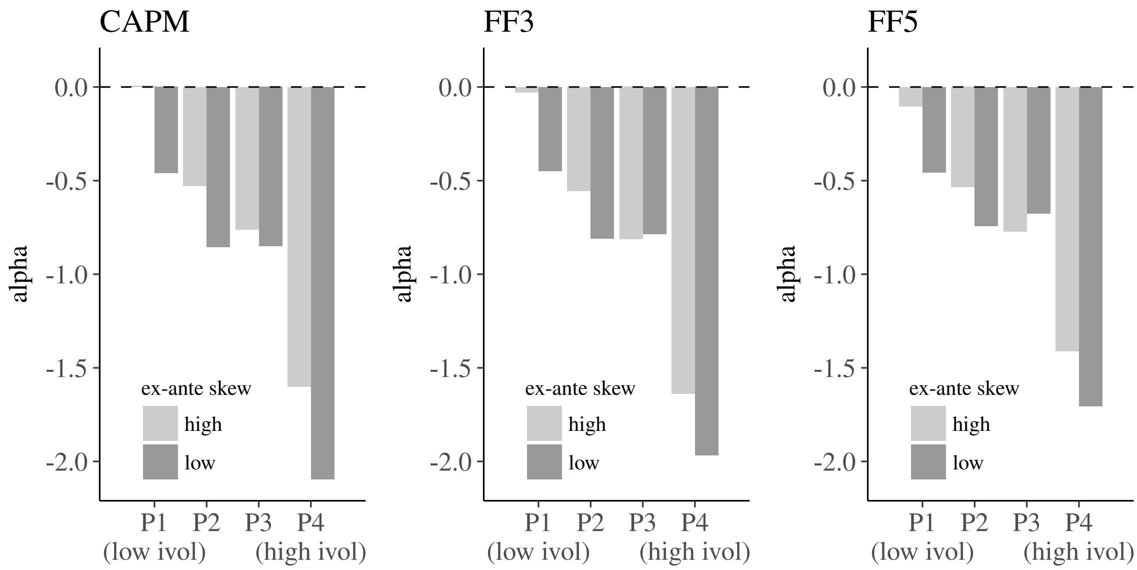

Figure (11) Double sorted portfolios upon ATM implied volatility and skewness

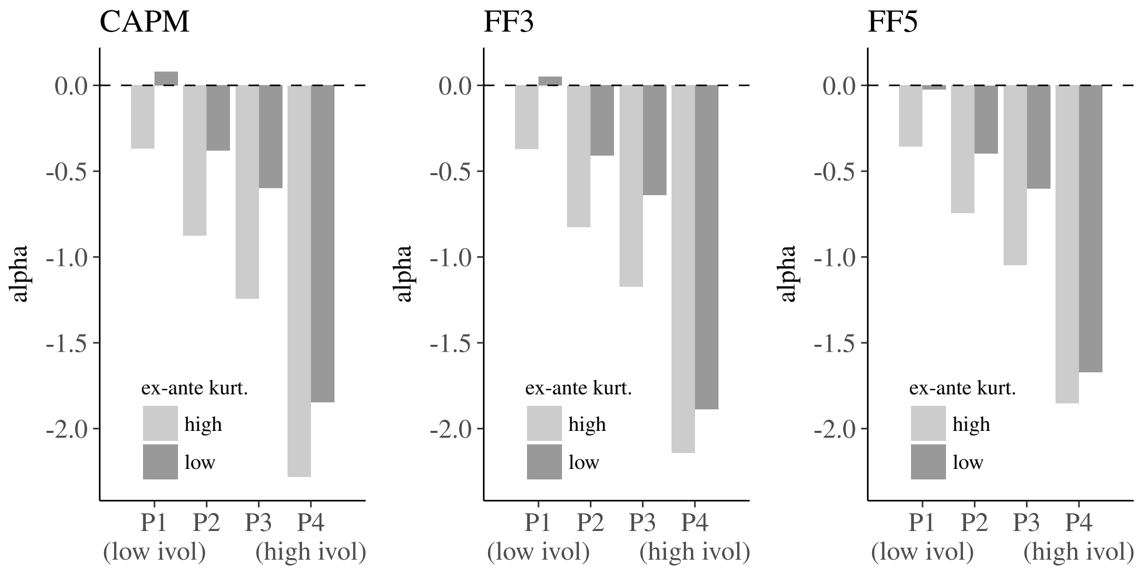

Figure (12) Double sorted portfolios upon ATM implied volatility and kurtosis

To conclude this section, the risk anomaly is persistent no matter if portfolios were built on beta or upon ATM implied volatility. For both settings one can split up the portfolios into low and high ex-ante skewness and kurtosis, whereas always the portfolios differing more from normality generates more unfavorable intercepts. Within the next section, it is tried to merge together skewness and kurtosis to one single proxy for tail risk to get an even stronger pronunciation of the effect.

4.2.2 Triple Sorts

The triple sorting procedure is realized as follows. Re-balancing frequency is set on a monthly basis, where every month each stock's rolling beta plus ex-ante skewness and kurtosis are estimated. To make the triple sort comparable to the double sorts, I cluster the three measures each into quartiles. The outcome is represented in Fig. 13. As expected, the gap between low and high tail risk portfolios is wider than under double sorts only.

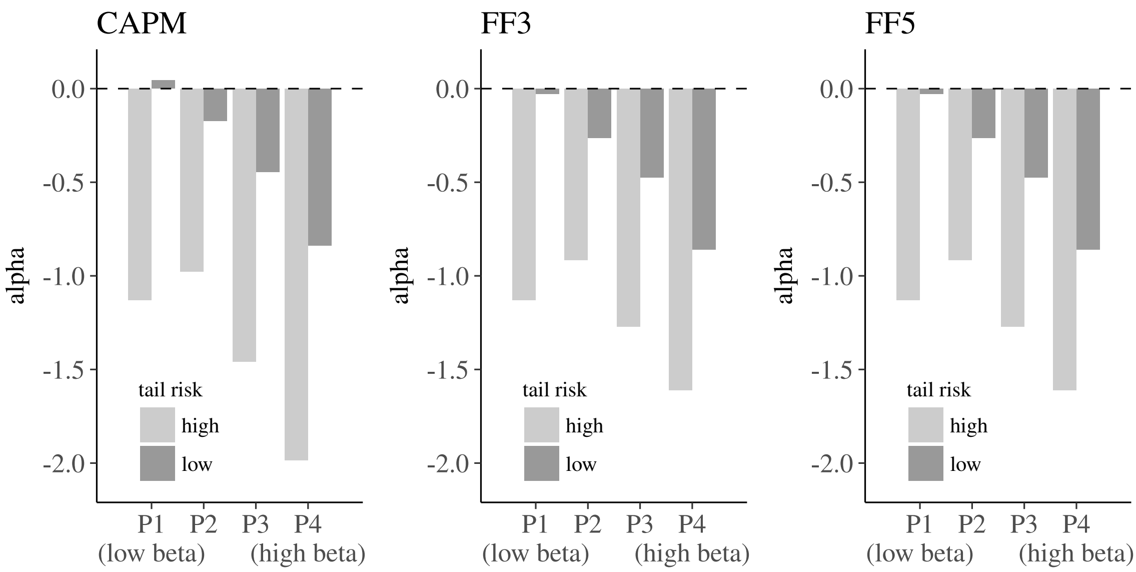

Figure (13) Triple sorted portfolios with on tail risk and beta

Table (12) Triple sorted portfolios on beta and ex-ante tail risk

	low ex-ante tail risk				*high ex-ante tail risk*			
	P1 L (low beta)	P2 L	P3 L	P4 L (high beta)	P1 H (low beta)	P2 H	P3 H	P4 H (high beta)
excess return	0.57	0.64	0.52	0.34	-0.43	-0.05	-0.38	-0.52
CAPM alpha	0.04	-0.17	-0.44	-0.84	**-1.13**	**-0.98**	**-1.46**	**-1.99**
	-0.23	(-0.63)	(-1.45)	(-1.7)	(-3.54)	(-3.46)	(-3.79)	(-3.79)
FF3 alpha	-0.03	-0.26	-0.47	-0.86	**-1.13**	**-0.92**	**-1.27**	**-1.61**
	(-0.16)	(-1.01)	(-1.58)	(-1.82)	(-3.56)	(-3.44)	(-3.79)	(-3.95)
FF5 alpha	-0.11	-0.3	-0.43	-0.62	**-1.14**	**-0.91**	**-1.23**	**-1.33**
	(-0.6)	(-1.14)	(-1.46)	(-1.35)	(-3.61)	(-3.4)	(-3.66)	(-3.5)
skew (ex-ante)	-0.23	-0.23	-0.23	-0.23	-0.35	-0.36	-0.36	-0.36
kurtosis (ex-ante)	6.05	5.51	5.28	3.09	11.76	11.84	11.8	11.86
beta (ex-ante)	0.64	1	1.26	1.63	0.7	1.01	1.26	1.6
beta (realized)	0.65	1	1.2	1.46	0.87	1.14	1.34	1.81
volatility	15.26	22.92	27.02	35.29	21.61	25.62	30.98	42.02

significant alpha values (p-value $< 5\%$) are displayed in bold

Triple sorting allows an even better distinction for smart beta strategies in both absolute and relative performance. Fitting the entire picture found so far, significance levels are again higher for those portfolios related to superior short selling costs, aka high beta and high kurtosis. Drawback of the independent triple sorting is that the number of stocks captured within each portfolio is reduced, thus portfolios may suffer from insufficient diversification, but as the observed stock set is quite broad, every portfolio contains on average above 100 stocks.

Anyway, I find clear evidence that betting against beta plus tail risk is able to produce superior alphas than betting against beta alone. Risk-adjusted intercepts are close to zero for low beta portfolios whereas negative for high beta portfolios. In absolute performance, the profitability of beta strategies can be clearly enhanced when betting against ex-ante kurtosis. For all levels of beta, the excess return for low kurtosis firms is higher than for high kurtosis opposites. This finding is in line with the 'distress puzzle' (cp. Campbell et al. (2008)), the empirical results derived do not indicate any compensation for the additional credit risk taken. Using the ex-ante measures for skewness and kurtosis allows to create strategies which are able to bet against significant asset pricing anomalies. In order to fully monetize this anomaly, investors need to short-sell high-beta with high-tail risk stocks and go long in low-beta low-tail risk. This Sharpe-ratio maximizing strategy comes with two main problems, (i) first it is difficult to form a portfolio that is immune against the systematic risk component since beta varies over time, and (ii) second, short selling high beta sounds easy in theory but comes with a difficulty in real world application, which could be one possibly reason why the beta and the tail risk anomalies still exist (cp. Baker et al. (2011)). However, performance of related strategies is of interest, so I set up a Sharpe-ratio maximizing performance back-test for the betting against beta and betting against beta plus tail risk to compare the profitability of beta and tail risk aversion.

Long-Short portfolio. From the results found so far, I see the betting against beta (BaB) strategy most profitable, if investors do not only trade against the market risk, but also versus ex-ante tail risk (BaB+TR). This means, BaB is assumed to work best if investors go long into stocks with low beta and low tail risk and short sell those stocks with high beta and high tail risk. Such a strategy can be implemented by hedge funds, building a beta-neutral portfolio to monetize the significant mispricing effect. I test for such a strategy by using rolling beta estimates, monthly re-balancing and rolling ex-ante tail risk. Each month, I calculate every stock's market risk from 2-year historical beta and the ex-ante skewness as well as kurtosis from the non-parametric estimator. The stocks are then independently clustered into quartiles by the three measures, where skewness and kurtosis again represent the tail risk. The stocks from the low beta, high skewness and low kurtosis quartiles are assigned to the long positions (l) of the portfolios, where all relevant stocks are value weighted to each other. Stocks falling into the highest beta, lowest skewness and highest kurtosis quartile will be assigned to the short positions (s), also here, the stocks are weighted according to their market capitalization such that the weightings sum up to one. From this schematic, the return series of the long portfolio and the short portfolio are computed,

$$r_L = \sum_{i}^{L} w_{i,l} \cdot r_{i,l} \tag{25}$$

$$r_S = \sum_{i}^{S} w_{i,s} \cdot r_{i,s}, \tag{26}$$

with L (S) as a vector of all stocks i from the long (short) portfolio, $w_{i,l}$ ($w_{i,s}$) representing each stock's weighting in the long (short) portfolio, and $r_{i,l}$ ($r_{i,s}$) as the stock returns.

At the beginning of every month, the BaB portfolio uses those long and short portfolios to take on a beta neutral position, considering the portfolio beta β_{PF} is proxied by

$$\beta_{PF} = w_L * \beta_L + w_S * \beta_S \tag{27}$$

and setting the constrain that the weighting of the long portfolio w_L plus the short portfolio w_S have to sum up to one,

$$w_L + w_S = 1. \tag{28}$$

Following the two equations and setting $\beta_{PF} = 0$ to enter the beta neutral positions, the portfolio weightings are calculated as

$$w_S = \frac{-\beta_L}{\beta_S - \beta_L} \tag{29}$$

and

$$w_L = 1 - w_S. \tag{30}$$

Note that through this framework, w_S takes on a negative value, indicating the short selling of that portfolio. With the two weightings determined, the BaB portfolio return

r_{BaB} is then simply

$$r_{BaB} = w_L * r_L + w_S * r_S. \tag{31}$$

Table 13 and Fig. 14 give insights into the performance analysis of betting against beta (BaB), betting against beta and tail risk (BaB+TR), long-only portfolio on low beta (long BaB) and long-only portfolio of low beta plus low tail risk (long BaB+TR). I denote a clear out-performance of the tail risk considering strategies above their low beta only counterparts in both, long-short and just long portfolios. Table 13 presents each, the analysis of the long-short portfolio (PF), the long portfolio (long) as well as the shorted stocks (short) of BaB and BaB+TR respectively.

Table (13) Comparison of betting against beta (BaB) and betting against both beta and tail risk (BaB+TR)

	BaB			*BaB+TR*		
	PF	long	short	PF	long	short
excess return	0.88	0.47	-0.03	1.29	0.59	-0.37
CAPM alpha	**0.83**	-0.07	**-1.26**	**1.32**	0.08	**-1.77**
	-2.47	(-0.67)	(-5.17)	-2.78	-0.48	(-3.86)
FF3 alpha	**0.73**	-0.1	**-1.18**	**1.17**	0.05	**-1.6**
	-2.52	(-1.04)	(-5.7)	-2.9	-0.32	(-4.36)
FF5 alpha	0.43	-0.17	**-0.95**	**0.87**	-0.03	**-1.3**
	-1.74	(-1.94)	(-5.4)	-2.3	(-0.18)	(-3.8)
beta (realized)	0.06	0.71	1.60	-0.04	0.65	1.81
volatility	14.6	14.16	32.11	20.6	14.5	39.61
skewness	-0.34	-0.3	-0.51	-0.27	0.04	-0.14
kurtosis	10.13	13.04	11.02	9.5	14.84	11.14
PTR	0.02	0.01	0.01	0.44	0.38	0.17
Sharpe ratio	0.72	0.40	-0.01	0.75	0.49	-0.11

significant alpha values (p-value $<$ 5%) are displayed in bold

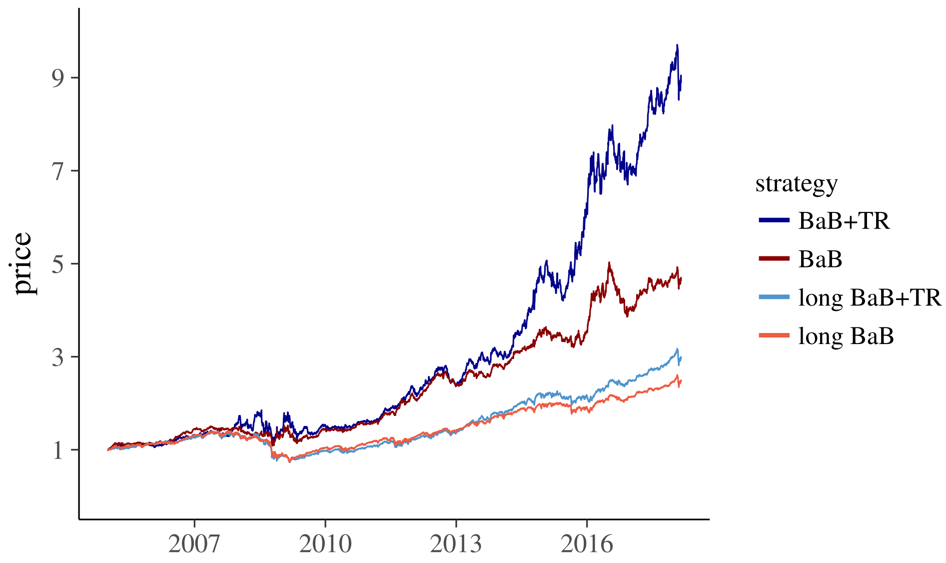

Figure (14) Price development of the two strategies before transaction costs

Equally to Schneider et al. (2016), the performance of betting against beta strategies can be enhanced by additionally betting against tail risk. The beta-neutral portfolio of BaB+TR generates roughly 1.5 times higher excess return than BaB (1.29% monthly vs. 0.88%). This gap is even wider when looking on realized alphas where the difference is 0.49% monthly. For both long-short strategies, realized alphas evolve to be significant at a level below 5% within CAPM, FF3 and FF5, except the BaB alpha tested under FF5. Also measured in terms of annualized Sharpe ratio, BaB+TR is with 0.75 higher than BaB facing 0.72. What can also be clearly seen is how the Sharpe ratio drastically increases when high beta stocks are shorted in addition to the long-only positions. When analyzing the ability of BaB and BaB+TR in taking on beta neutrality (beta = 0), it can be observed that on average they did quite well with mean realized betas of 0.06 and -0.04 respectively. Since beta is not constant over time, realizing a portfolio beta of exactly zero is kind of impossible. To examine the beta realized by the long-short strategies, a rolling computation is included in Fig. 15. While the mean realized beta comes close to zero, Fig. 15 indicates that monthly rebalancing is not frequent enough to be really beta-neutral. As realized beta ranges from ± 0.4, volatilities evolve themselves to be quite high for a so-called beta-neutral strategy.

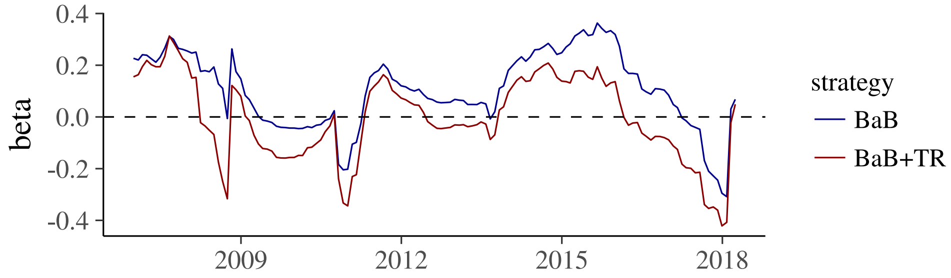

Figure (15) Rolling realized beta

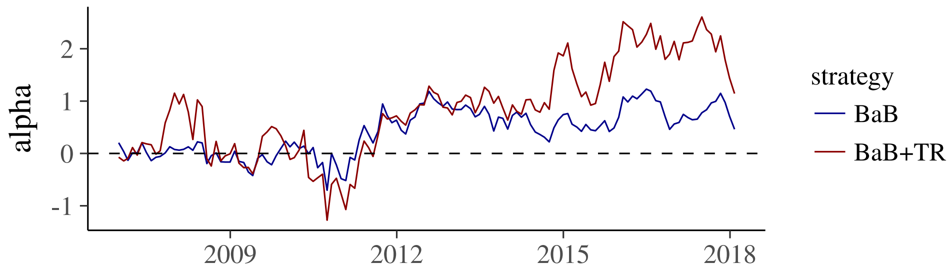

Figure (16) Rolling realized alpha (FF5)

Fig. 16 uncovers that especially since the end of 2014 additionally betting against tail risk could distinguish the performance above long-short BaB. Also interestingly, while realized alphas of the two strategies were moderate and around zero before 2012, after then they showed strong ability in outperforming after risk-adjustment. Betting against tail risk in smart beta strategies is not only profitable when short selling is necessary, Table 13 gives also insights of how the long and short positions themselves performed. As conclusion, avoiding high tail risk stocks in the low beta long positions increases excess return by additional 1.44% yearly at roughly equal volatility levels, whereas also realized alpha is higher by 1.8% percentage points a year. This highlights the importance of tail risk in long-only strategies.

In the result brought from the back-test, no short-selling fees nor transaction costs were considered. Estimating those costs relies on several assumptions, for example large institutional investors are able to trade securities on lower prices than small investors. Therefore, at this point, to conclude a rough idea of net profitability a simplified cost approximation is made. I compute portfolio turnover rates according to the U.S. SEC equally as in Section 4.1.1. With clustering the stock universe into quartiles of beta, the categories in BaB show to be surprisingly consistent with only small sized firms dropping out and bought back in inducing very low portfolio turnover rates. For BaB+TR the PTR are way higher but still generally low. Hence from the side of trading costs I do not see strong impact on the net profitability of both strategies. Bogle (2007) suggests trading costs to be approximated by 1% of the portfolio's turnover rate. Making assumptions about the net profitability of the long-short strategies further requires extensive research on the restrictions, realizability and costs of short selling for all stocks and over the entire time horizon, thus at this point I make simple assumptions based on the results of D'Avolio (2002), who settles his empirical research about short selling upon the same data source (CRSP). He concludes that less than 1% of the entire CRSP market capitalization is covered by stocks that cannot be sold short, hence from this side the realizability of - in this case value weighted - BaB and BaB+TR should be plausible. Difficulties arise when correctly assigning short selling costs. While easy-to-borrow stocks representing 91% of market capitalization face stock loan fees below 1% per annum, there are roughly another 8% of stocks with short selling fees around 4%, but high bias potential is induced by the very hard to borrow stocks that afford fees of up to 50% a year. This diversity makes it hard to make an flat statement on general short selling costs. But due to simplicity, I apply an fee of 25 basis points per annum for all stocks which is estimated by D'Avolio (2002) as the value-weighted mean loan fee for the U.S. stock market. The transaction cost estimation is attached in Appendix C. BaB+TR realizes a higher turnover ratio, however, with the triple sort it requires less stocks to short-sell, therefore loan fee is less for that strategy compared to BaB only. Considering both trading and short-selling costs, betting against tail risk always pays off, no matter if investors follow long-only or long-short styles.

For example, the long-BaB+TR portfolio realizes with 6.72% yearly net excess return 1.08 percentage points more than the betting against beta comparable at approximately equal volatility levels of 14.5%. Both beta-neutral strategies evolve themselves to realize economical significant alphas, also after costs, where betting against tail risk could largely increase the risk-adjusted returns above the BaB style. Consequently, also after costs betting against tail risk is a profitable approach to further push performance of smart beta strategies and comes with the nice feature that it is quite easy to implement. Anyway, the costs approximated here are a simplified and very rough estimation, real transaction costs may vary largely from those suggested here, but still, I strongly believe that betting against tail risk is highly profitable.

To summarize this chapter, using conditional double and triple sorts show, that the beta anomaly can be further split up into a skewness and kurtosis anomaly. The results found are in line with Schneider et al. (2016), ex-ante skewness positively predicts stock returns, for ex-ante kurtosis the relation is negative and thus support for Bali, Hu & Murray (2017). Analyzing strategy performances I conclude betting against beta strategies to increase in profitability when investors trade against ex-ante tail risk. Linking ex-ante tail risk to credit risk, I observe the same persistence of the 'distress puzzle' as found by Campbell et al. (2008).

5 Tail Risk Asset Pricing

As the empirical survey proofs clear structures of ex-ante tail risk occurrence in relation to stock's excess returns, Schneider et al. (2016) further claim that the beta anomaly can be resolved by accounting for tail risk, I therefore create an own asset pricing factor trying to catch tail risk and compare its relevance directly to the explain-ability of common Fama-French factors. There are two main requirements on the self created asset pricing model, (i) the overall capability of expressing portfolio returns has to be higher than at existing asset pricing models while keeping high significance of the used factors and (ii) both anomalies should be at least partially removed. I construct the tail risk factor following the suggestion of Fama & French (1993), forming portfolios on certain fundamentals. In detail, the factor is defined as the average return of the high tail risk portfolio minus low tail risk portfolios, representing ex-ante tail risk. Or in other words, the ex-ante tailed portfolio minus normal-distributed portfolio (='TMN'). The TMN factor is then calculated as follows:

$$TMN = \frac{1}{2} * (TailRisk) - \frac{1}{2} * (NormalRisk) \tag{32}$$

The two portfolios are formed on a value weighting base according to the triple sorting technique used in Section 4.2.2. Every month, the stock universe is categorized fifty-fifty once by ex-ante skewness and independently on ex-ante kurtosis. The stocks matching low skewness and high kurtosis are assigned to the tail risk portfolio - those with high skewness and low kurtosis to the normal risk portfolio, see Table 14.

Table (14) Definition 'Tail Risk' and 'Normal Risk'

		kurtosis	
		low	high
skewness	low		Tail Risk
	high	Normal Risk	

Constructing the factor, Table 15 gives a short summary of the TMN factor, the tail risk (T) and the normal risk (N) portfolio.

Table (15) Summary statistics of the TMN factor

	TMN	N	T
return [p.a.]	-5.66	9.38	-1.93
volatility	11.07	16.96	31.26
beta realized	0.34	0.81	1.48

Now that the tail risk representing factor is constructed and computed, a relative analysis of the factor in combination with the other asset pricing models is initialized. Information about the distribution of the TMN factor is attached in Appendix D.

5.1 Tail Risk and CAPM

The evidence for significant impact of skewness and kurtosis on asset prices is clear. I initially stated the hypothesis, that implementing ex-ante tail risk into asset pricing models would at least partially remove the beta anomaly. From the quantitative analysis before I came to the conclusion, that tail risk indeed releases strength of the beta anomaly when ex-ante non-normality is considered. Therefore, an own tail risk capturing asset pricing factor (TMN) following Fama & French (1993)'s factor development concepts was created. After inventing the TMN factor, I define a new, CAPM extended asset pricing model 'CT' noted as follows:

$$CT: \quad r_i - r_f = \alpha + \beta_0 \cdot (r_{mkt} - r_f) + \beta_1 \cdot TMN \tag{33}$$

Where r_i indicates a portfolio i's return, r_f the risk-free rate and r_{mkt} the market return. β_0 will represent the portfolio's loading of systematic risk exposure, β_1 the tail risk level. Please note that when speaking of beta in this reading, generally the systematic market risk aka CAPM coefficient β_0 is meant.

Having defined the TMN factor as well as the CT model, three main hypothesis are tested which arose at the beginning of modeling the new asset pricing framework.

Hypothesis 1: By accounting for tail risk, the mispricing of high beta stocks should be less anoumalous than under CAPM, FF3 and FF5.

Hypothesis 2: Since it is assumed that all available information is priced in at the market for every point of time, I state the hypothesis that option prices reflect information which is also covered by FF5 factors. Hence in-cooperating ex-ante tail risk should roughly deliver an equally good fitting asset pricing model, such that by introducing the tail risk factor, the other FF factors will be no longer needed giving a model with equally statistical power but with less factors required.

Hypothesis 3: As it is shown in Section 4 that tail risk alone could not entirely resolve the beta anomaly, also the factor for lottery demand from Bali, Brown, Murray & Tang (2017) is introduced to test a three factor asset pricing model consisting of market beta, tail risk and lottery demand in order to produce a clearer, economically sounding and better fitting model than the three common models tested (CAPM, FF3 and FF5).

Testing H1: TMN more robust against beta anomaly

Addressing Hypothesis 1, I go back to the beginning of the empirical evaluation - the intital tests of the beta anomaly - and introduce a new model implementing the TMN factor. This new model is simply an extension of basic CAPM model by the TMN coefficient (CT). Therefore, while the Fama & French (1993, 2015) models have two or four additional factors, the CT model tries to deliver better explanation by solely one extra factor (TMN). In terms of empirical research, the same beta-only sorted portfolios are built and tested in the three common asset pricing frameworks, but now also in comparison with the tail risk extended CT model. Fig. 17 delivers insights of how the ten portfolio alphas in monthly 'risk-free' return (P1 = low beta, P10 = high beta) were found.

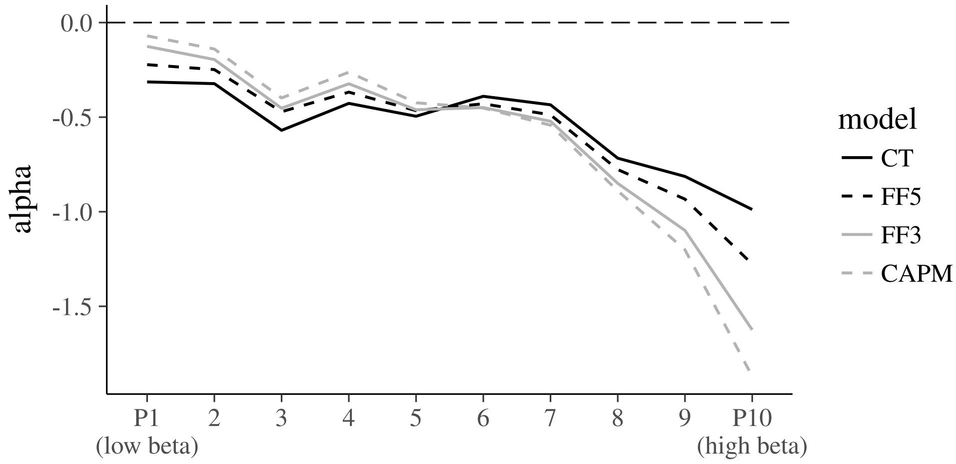

Figure (17) Capturing the beta anomaly: common models vs. CT

As result, the CT model leads into a flatter relation between the realized alphas on beta-built portfolios beneath all of the four tested models. The picture is quite obvious, under CAPM the beta anomaly is pronounced the strongest. By adding the Fama-French factors the line gets flatter and flatter. With the CT model, a framework is created which delivers the closest to zero slope of all four models, meaning that the difference in alpha for low and high beta portfolios is the smallest under CT. Hence, the CT model based on solely two factors was better able to remove the beta anomaly than FF5. From this result I state Hypothesis 1 to hold and confirm the statement of Schneider et al. (2016) that accounting for tail risk softeners the low risk anomaly.

Given confirmation of Hypothesis 1, the TMN factor appears to be better able in removing the beta anomaly than the Fama-French factors. However, since the conclusion is derived from the results tested in ten beta built portfolios, I catch up with the bootstrapping methodology to answer questions about the robustness of Hypothesis 1. Doing so, the asset pricing models are tested in 1000 portfolios where stocks are randomly selected and weighted according to their market capitalization. Each of the portfolios is well diversified with a number of stocks included between 300 and 1500. Quantifying the severity of the beta anomaly, the realized alphas together with realized betas are computed. Fig. 18 shows the output, strongly supporting Hypothesis 1. In direct comparison of FF5 with CT, the later model delivers way smaller alpha-beta correlation realizing a flatter fitted line and less significant alphas (cp. Table 16).

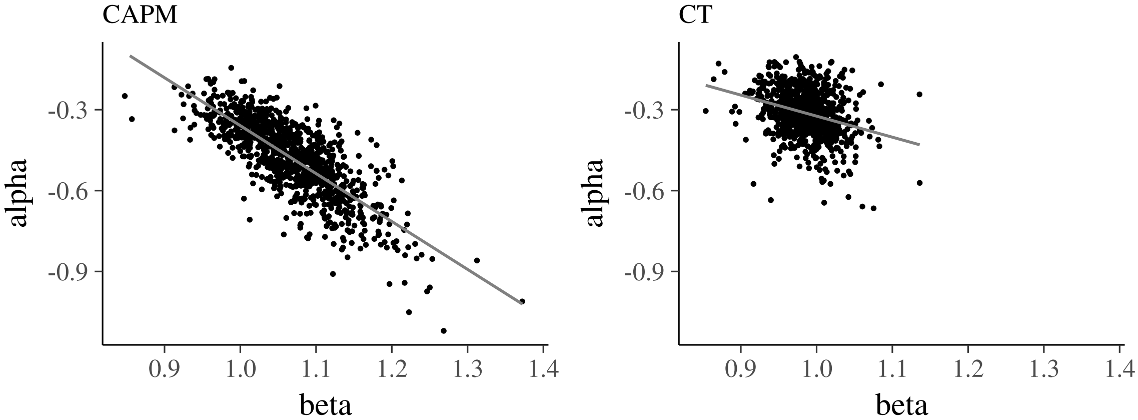

Figure (18) Beta Anomaly: Hypothesis 1 is robust. CT model creates less $\alpha - \beta$ correlation and closer to zero mean than CAPM.

Table (16) Pearson's correlation between alpha and beta as a quantification of the beta anomaly

	CAPM	FF3	FF5	CT
$Cor(\alpha,\beta)$	-0.80	-0.72	-0.62	-0.28

Defined in this work, the size of the beta anomaly can be proxied as the correlation of alpha with beta, Table 16 nicely shows how that correlation fades away among the models. For example CAPM shows strong existence of the beta anomaly with a correlation of -0.80, whereas the TMN extension removes large portions of that link to a level of -0.28.

Not only that the CT model better resolves the beta anomaly than the other compared models, it also shows high validity through significance of its explanatory variables, see Table 17. The CT model had in 96.6% of the 1000 analyzed portfolios both explanatory factors highly significant, while the Fama-French factors evolved to be far less significant. What is more, finance theory suggests alpha values to be insignificant. Since on general all models produced rather high significance levels for the intercepts, CT turned out to be most able reducing that significance down to 53.9%. Table 16 can be interpreted as the coefficients percentage producing p-values below the significance barrier (0.1%) out of the 1000 regressions run.

Table (17) Percentage of the factor's p-value below 0.001 from the bootstrapping analysis; FF models are less significant than the CT model

	p-value <0.001 (in %)			
	CAPM	FF3	FF5	CT
alpha	69.4	92.1	92.0	53.9
beta	100	100	100	100
SMB		61.9	64.7	-
HML		97.5	95.3	-
RMW			63.7	-
CMA			69.0	-
TMN				96.6

Testing H2: Goodness of fit

Schneider et al. (2016) can be interpreted that the multi factor models use more factors that are actually necessary. Hints in that direction are already observed when tested for Hypothesis 1. With less significant alpha values and higher significance of the factors used, the CT model seems to fit similarly or equally good as the FF5 model with the side effect, that there are solely two factors instead of five. In order to consolidate Hypothesis 2, the goodness of fit among the models is analyzed. Generally speaking, between two models with equally goodness of fit, the one with less explanatory variables should be preferred. In this analysis, the models' goodness of fit are quantified by the adjusted R squared,

$$R^2_{adj} = 1 - \frac{(1-R^2)(n-1)}{n-k-1}, \tag{34}$$

denoting n as the number of data points, k the number of input variables and R^2 is simply the standard goodness of fit measure. By using R^2_{adj}, the models can be directly compared as the fitting inflation effect from adding extra factors is corrected. The overall R^2_{adj} is very high, which can be taken from Fig. 19. Even the simplest model -

CAPM - is able to explain the data at an median R^2_{adj} of 95.7%. The more sophisticated models only slightly increase that measure, where the other three models have almost equally R^2_{adj} at levels of 96.4%, 96.5% and 96.7% for the CT, FF3 and FF5 respectively. As those measures lie really close together, I claim the three models to have equal capability in fitting the return data.

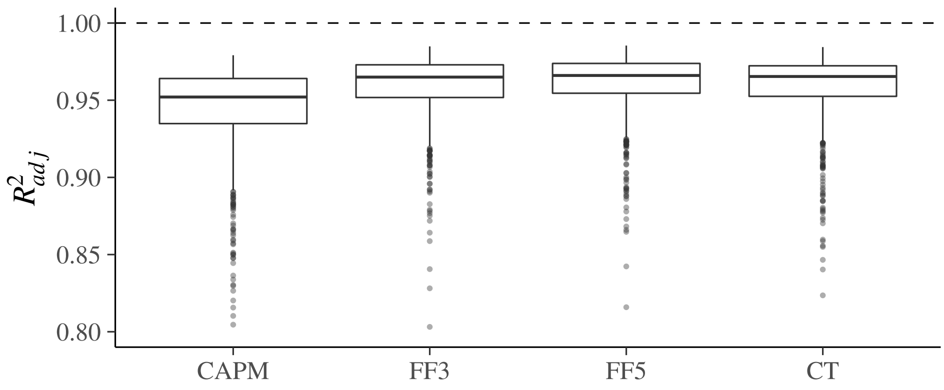

Figure (19) Goodness of fit is generally very high for all tested models.

From Fig. 19 I do not really observe an improvement in the goodness of fit when changing from FF3 to FF5, arising the possibility that the extra two factors in FF5 do not necessarily add extra explain-ability. Using CT allows to achieve the same fitting quality with only two factors, thus I find confirmation for Hypothesis 2, meaning that the two factor model makes the FF5 factors redundant. Given Hypothesis 2 holds, I believe the CT model to be a more suitable asset pricing model than the FF5 when it comes to the goodness of fit. Anyway, correlation between alpha and beta is given and other argumentation of that anomaly is sourced in lottery demand. Consequently, empirical tests on the lottery demand factor and combinations with TMN are made.

5.2 The CTF model

Accounting for non-normality expectations in asset pricing adds quality to the models and resolves the beta anomaly partially. However, with the empirical testing of the

CT model I still find evidence for the beta anomaly to persist such that it cannot be entirely removed by the CT model. Bali, Brown, Murray & Tang (2017) introduce the FMAX factor which allows to account for the lottery demand effect of high beta stocks. This measure is built following the idea of Frazzini & Pedersen (2014) that one main driver of the beta anomaly is the investor's unnatural high demand for high beta stocks, as those are suggested to realize higher absolute returns. To test for Hypothesis 3, I implement the idea of investor's lottery demand and extend the CT model by Bali, Brown, Murray & Tang (2017)'s FMAX factor computed from the own dataset.

The FMAX factor is based on Frazzini & Pedersen (2014)'s MAX measure. A stock's MAX is defined as the sum of the five highest daily returns of a month, so for every stock in the analysis, monthly MAX proxies are computed. To create the lottery demand factor FMAX itself, I follow Fama & French (1993)'s factor forming technique. Each month, stocks are sorted according to their MAX value, then divided into two clusters so that there are two categories containing equally number of stocks: low-MAX and high-MAX. The low-MAX stocks are value weighted against each other to form a portfolio, same procedure is done to form the high-MAX portfolio. In the last step, Bali, Brown, Murray & Tang (2017)'s daily FMAX factor is estimated as the average returns of the high-MAX portfolio minus the low-MAX portfolio,

$$FMAX = \frac{1}{2} \cdot highMAX - \frac{1}{2} \cdot lowMAX. \tag{35}$$

Table (18) Summary of the FMAX factor

	FMAX	lowMAX	highMAX
excess return [p.a.]	-6.26	6.57	-3.63
volatility	9.76	16.51	31.63
beta realized	0.33	0.86	1.52

With the estimates of the lottery demand factor, the CT asset pricing model is put into new perspectives. I want to know how much influence the FMAX has in resolving the beta anomaly and how CAPM extended by TMN and FMAX will perform. Thus, testing for Hypothesis 3, I define the CTF model as follows:

$$CTF: \quad r_i - r_f = \alpha + \beta_0 \cdot (r_{mkt} - r_f) + \beta_1 \cdot TMN + \beta_2 \cdot FMAX \tag{36}$$

To allow comparison as well as interpretation of each factor's reliability and effect, a CF model is defined existing of CAPM extended by FMAX only

$$CF: \quad r_i - r_f = \alpha + \beta_0 \cdot (r_{mkt} - r_f) + \beta_1 \cdot FMAX, \tag{37}$$

so that the TMN factor can be directly compared with the FMAX factor, making conclusions about usefulness of the three factor CTF model more plausible.

Information upon the distribution and one sample t-test on TMN and FMAX factors can be found in Appendix D Fig. 26 and Table 26. A short remark, both factors turn out to be significantly negative with 95% confidence intervals of -Inf. to -0.62 (-0.66) for TMN (FMAX respectively).

5.3 Model Comparison

As shown so far, the CT model results in a less correlated alpha-line when compared to FF5. In Fig. 20 the same analysis on beta-sorted portfolios is run to derive conclusions about FMAX extension.

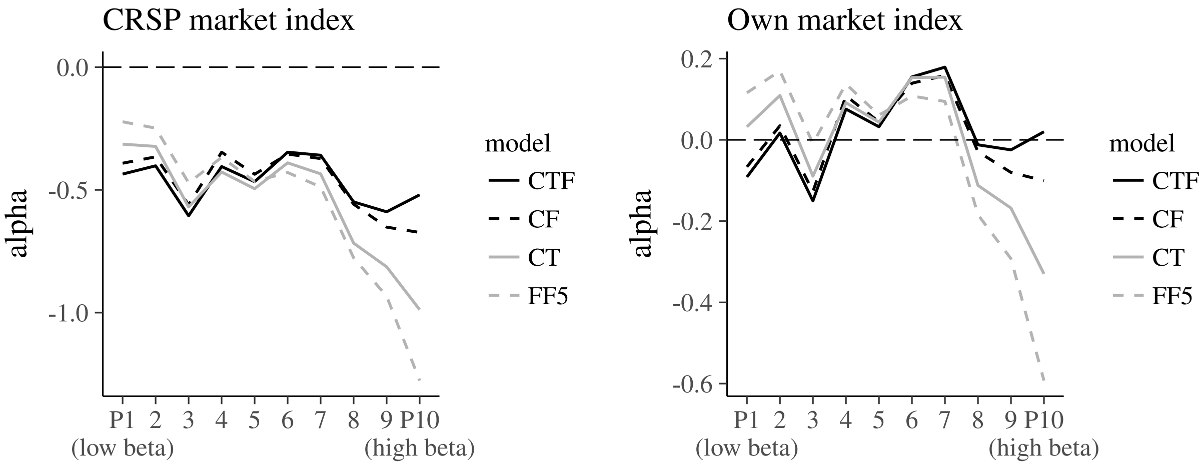

Figure (20) Realized alphas among beta-sorted portfolios

Table (19) Model comparison tested on the 10 beta sorted portfolios

	CAPM	FF3	FF5	CT	CF	CTF
$Cor(\alpha, \beta)$	-0.94	-0.91	-0.88	-0.80	-0.57	-0.27
avg. alpha error (CRSP index)	0.58	0.56	0.52	0.51	0.44	0.43
avg. alpha error (own index)			0.16	0.12	0.08	0.07

What can be seen in this plot is that the FMAX factor removes the beta anomaly even stronger than the TMN factor does. The CTF model turns out to be most immune against beta correlation, but still, the two factor CF model comes very close to the three factor CTF. Notably, all of the three models (CT, CF, CTF) are less affected by high beta mispricing than the Fama-French 5 factor model. Table 19 shows that alpha values are closest to zero - as assumed in finance theory - within the CTF model. Average alpha errors are calculated as mean absolute errors. The correlation values in that table should be taken with caution as they simply measure the link between the 10 datapoint pairs of portfolio's alpha and beta. Nonetheless, it indicates what can also be seen by visual inspection of the alpha-beta plot Fig. 20, the CTF model turns out to be most robust against the beta anomaly.

That the CTF model is of high significance is proven in the regression summary of

Table 20, significant coefficients (p-value $< 5\%$) are highlighted in bold. All of the four compared models - FF5, CT, CF and CTF - turn out to realize high significance for every regression coefficient. Table 20 gives detailed insights of the asset pricing model's outputs on testing the ten conditionally beta-sorted portfolios. The TMN factor can be interpreted straight forward - the lower the factor, the smaller are tail risk loadings in that portfolio. Same is true for FMAX, negative values reflect demand for safety, positive values indicate demand for lottery stocks. As suggested, the TMN and FMAX factors are superior for high beta portfolios and negative at low beta, therefore indicating under-performance of high beta portfolios. Notably, FF5 also reflects some dependencies of its factors upon market beta. SMB and HML tend to positively relate with beta, for RMW and CWA the link looks negative.

In terms of model efficiency and unbiased coefficients, additional residual and multicollinearity analysis of the CTF regression model itself is attached in Appendix E. Generally speaking, residuals seem to follow a normal distribution with slight positive leptokurtosis. Variance of the residuals looks to be greater during the time of the financial crises in 2008, but besides, I do not observe any clear structure dependencies in residuals' variance, therefore I suggest homoskedasticity to hold. Also tests of predictor's variance inflation factors (VIF) and eigenvalues does not indicate multicollinearity. In conclusion, considering the high significance of each coefficient, I believe CTF to be an unbiased and efficient regression model.

Table (20) Model comparison: regression coefficients

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
					FF5					
alpha	-0.22	**-0.25**	**-0.47**	**-0.37**	**-0.47**	**-0.43**	**-0.49**	**-0.78**	**-0.93**	**-1.28**
	(-1.62)	(-2.21)	(-4.25)	(-3.04)	(-3.94)	(-3.2)	(-2.9)	(-3.91)	(-4.41)	(-4.18)
beta	**0.70**	**0.86**	**0.97**	**1.06**	**1.10**	**1.13**	**1.22**	**1.25**	**1.36**	**1.44**
	(-114.04)	(-173.29)	(-196.37)	(-197.23)	(-209.99)	(-189.02)	(-163.15)	(-141.87)	(-144.5)	(-106.2)
SMB	**-0.21**	**-0.08**	**-0.05**	**-0.06**	**0.02**	**0.10**	**0.13**	**0.21**	**0.19**	**0.16**
	(-17.27)	(-7.99)	(-5.39)	(-5.23)	(-2.23)	(-8.66)	(-8.62)	(-12.05)	(-10.25)	(-5.98)
HML	**-0.21**	**-0.19**	**-0.20**	**-0.18**	**-0.11**	0.01	**0.07**	**0.15**	**0.38**	**0.82**
	(-17.6)	(-19.69)	(-21.34)	(-17.32)	(-11.08)	(-0.79)	(-4.97)	(-9.02)	(-20.84)	(-31.52)
CMW	**0.22**	**0.13**	0.03	**0.13**	0.02	**-0.04**	**-0.08**	**-0.16**	**-0.39**	**-0.88**
	(-10.32)	(-7.47)	(-1.71)	(-7.05)	(-1.25)	(-2.15)	(-3.01)	(-5.2)	(-11.8)	(-18.71)
RWA	**0.46**	**0.19**	**0.17**	0.01	**-0.06**	**-0.10**	**-0.15**	**-0.39**	**-0.73**	**-1.06**
	(-20.06)	(-10.4)	(-9.21)	(-0.49)	(-3.14)	(-4.55)	(-5.47)	(-11.83)	(-20.71)	(-20.91)
					CT					
alpha	**-0.31**	**-0.32**	**-0.57**	**-0.43**	**-0.5**	**-0.39**	**-0.43**	**-0.72**	**-0.81**	**-0.99**
	(-2.13)	(-2.85)	(-5.16)	(-3.47)	(-4.11)	(-2.87)	(-2.56)	(-3.47)	(-3.63)	(-3.21)
beta	**0.7**	**0.87**	**0.99**	**1.07**	**1.11**	**1.12**	**1.22**	**1.26**	**1.36**	**1.38**
	(-101.25)	(-165.62)	(-191.58)	(-186.5)	(-197.75)	(-177.44)	(-153.62)	(-131.26)	(-129.89)	(-96.08)
TMN	**-0.31**	**-0.23**	**-0.22**	**-0.21**	**-0.09**	**0.08**	**0.14**	**0.22**	**0.5**	**1.13**
	(-26.84)	(-26.25)	(-25.22)	(-21.73)	(-9.6)	(-7.41)	(-10.31)	(-13.6)	(-28.09)	(-46.5)
					CF					
alpha	**-0.39**	**-0.37**	**-0.56**	**-0.35**	**-0.44**	**-0.35**	**-0.37**	**-0.56**	**-0.65**	**-0.67**
	(-2.71)	(-3.23)	(-4.87)	(-2.63)	(-3.56)	(-2.63)	(-2.22)	(-2.85)	(-3.06)	(-2.42)
beta	**0.73**	**0.89**	**0.98**	**1.03**	**1.08**	**1.11**	**1.19**	**1.2**	**1.29**	**1.26**
	(-101.43)	(-158.75)	(-171.23)	(-158.28)	(-178.3)	(-165.47)	(-143.01)	(-122.89)	(-122.4)	(-91.06)
FMAX	**-0.42**	**-0.29**	**-0.21**	**-0.11**	-0.02	**0.13**	**0.22**	**0.43**	**0.72**	**1.56**
	(-29.76)	(-26.68)	(-18.86)	(-8.53)	(-1.34)	(-9.66)	(-13.53)	(-22.57)	(-34.48)	(-57.49)
					CTF					
alpha	**-0.44**	**-0.4**	**-0.61**	**-0.4**	**-0.47**	**-0.35**	**-0.36**	**-0.55**	**-0.59**	**-0.52**
	(-3.11)	(-3.68)	(-5.52)	(-3.29)	(-3.88)	(-2.57)	(-2.14)	(-2.8)	(-2.84)	(-2.08)
beta	**0.75**	**0.91**	**1**	**1.06**	**1.1**	**1.1**	**1.18**	**1.19**	**1.26**	**1.18**
	(-104.98)	(-163.39)	(-179.38)	(-169.52)	(-179.69)	(-160.91)	(-138.92)	(-119.42)	(-119.87)	(-93.08)
TMN	**-0.18**	**-0.15**	**-0.18**	**-0.24**	**-0.12**	**0.03**	**0.05**	**0.04**	**0.25**	**0.62**
	(-13.42)	(-14.23)	(-17.27)	(-20.13)	(-10.71)	(-2.47)	(-3.46)	(-2.01)	(-12.77)	(-26.02)
FMAX	**-0.3**	**-0.19**	**-0.09**	**0.05**	**0.07**	**0.11**	**0.18**	**0.41**	**0.54**	**1.13**
	(-17.89)	(-14.89)	(-6.74)	(-3.73)	(-4.87)	(-6.63)	(-9.29)	(-17.57)	(-22.2)	(-38.44)

significant alpha values (p-value $<$ 5%) are displayed in bold

5.4 Robustness of the CTF model

Analyzing beta sorted portfolios, the beta anomaly strongly exists in CAPM but is bit by bit removed when applying more sophisticated asset pricing models. So far, the empirical analysis let me conclude, that the CTF model is best able to delete both non-zero regression intercepts as well as the beta anomaly itself. To accelerate the model robustness, it is analyzed in a bootstrapping framework with 1000 randomly sampled portfolios. Table 21 can be interpreted together with table 17, all coefficients turn out to gather small p-values with high percentages of meeting the significance levels. Please address Table 17 to compare the tail risk and lottery demand models with common CAPM, FF3 and FF5.

Table (21) Percentage of how often significance criteria (p-value $< 5\%$) is met in the 1000 analyzed portfolios

	p-value < 0.05 (in %)		
	CT	CF	CTF
	using CRSP market index		
alpha	92.5	97.4	91.4
beta	100	100	100
TMN	98	-	97.8
FMAX		94.9	69.8
	using own market index		
alpha	37.4	5.5	26.0
beta	100	100	100
TMN	97.7	-	99.6
FMAX		91.3	86.7

As mentioned earlier in Section 4.1, the systematic negative alphas are still present throughout all tested models. Anyway, since the analyzed data is reduced through certain criteria (market capitalization $>$ 300 Mio., options availability) plus corrected for survivorship bias but tested against the broader CRSP market index, it is plausible that the declared investment universe can not diversify as good as the used market benchmark causing significant negative alphas. Either way, alphas' significance disappears when the own market index - meaning the value weighted portfolio built out of the dataset - is used instead of the CRSP reference. Nonetheless, under both market references, regression coefficients for CT, CF and CTF realize higher significance levels than the FF5 factors (cp. Table 17).

Comparing goodness of fit, the picture looks pretty much the same as before. The CTF model captures same fitting ability as the Fama-French 5 factor model. Within this confrontation, the CTF model can be distinguished from CF given that its R^2_{adj} lies in (slightly) lower patterns with an average of 0.95 compared to 0.96 at CTF. The very high realized R^2_{adj} of the 1000 tested portfolios together with the overall high coefficient significance support robustness of the CTF model.

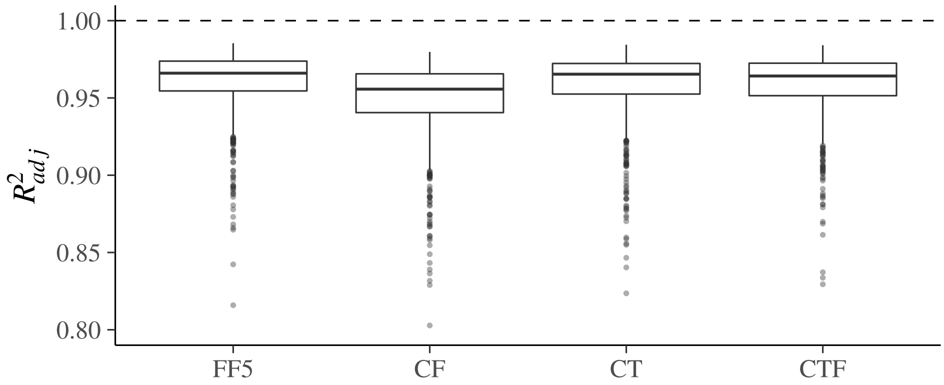

Figure (21) Comparison of adjusted R^2

Visualizing the beta anomaly in certain models. All in all, asset pricing models are confronted with the mispricing of high beta stocks. Within the last lineup, realized alphas vs. betas tested under CTF are plotted in Fig. 22, supplementary the linear fits of several other models are displayed. The flatter the fitted line, the less are alphas affected by beta and the greater the model's capability to remove the beta anomaly.

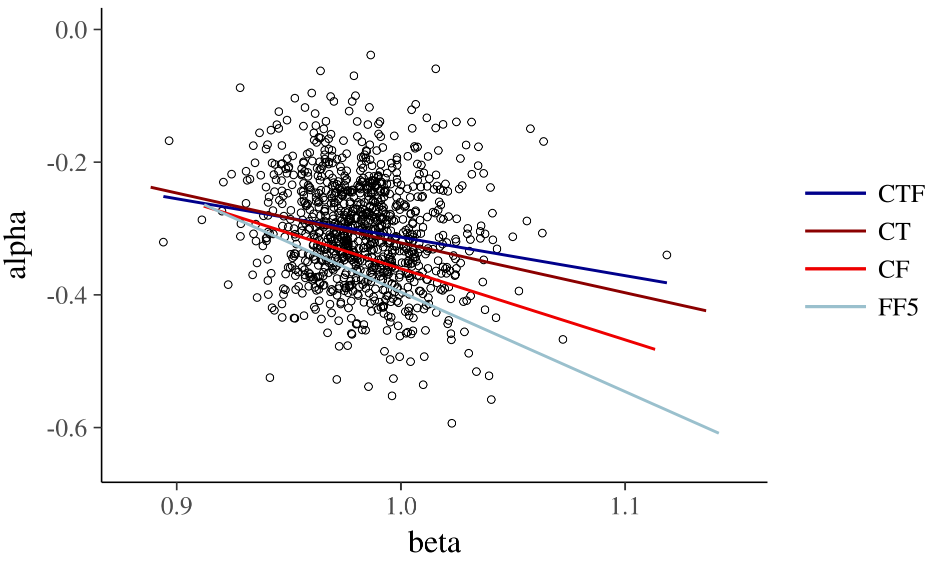

Figure (22) Points indicate alpha vs. beta from the bootstrapping results under CTF model, additionally linear fitted lines from other models are displayed to show that CTF realizes the flattest line.

Fig. 22 nicely shows that the CTF performs better than all other models when it comes to resolving the beta anomaly. While FF5 does worst in this task with the steepest fit, the tail risk representing TMN factor has superior power in removing beta dependence than the lottery demand factor FMAX by Bali, Brown, Murray & Tang (2017) has. In combination, CAPM extended by TMN and FMAX (CTF) looks quite effective in removing the dependence of 'risk-free' performance to systematic risk exposure. Now that 1000 realized alpha-beta pairs are gained, a correlation test between each model's intercept with the market risk factor can be interpreted more reliable than under conditional portfolio sorting. Table 22 delivers the results therefore. Using bootstrapping methodology, the strength of the beta anomaly is quantified by measuring the linear dependence of produced alphas with betas from 1000 different random portfolios, compared under several models.

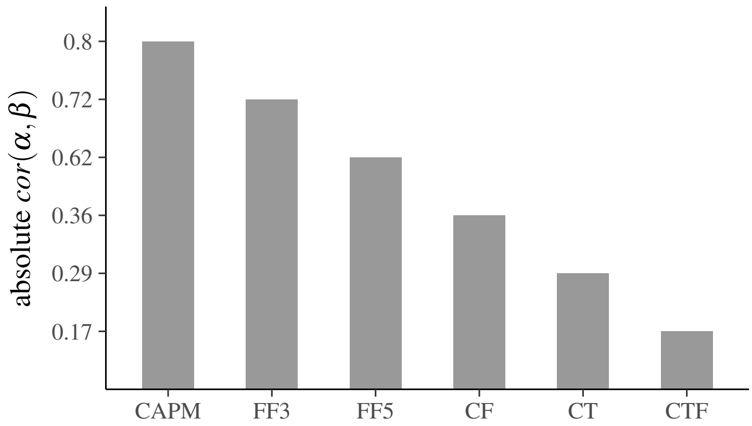

Figure (23) Absolute correlation between alpha and beta in the different models

Table (22) Quantifying the beta anomaly

	CAPM	FF3	FF5	CF	CT	CTF
$Cor(\alpha,\beta)$	-0.80	-0.72	-0.62	-0.36	-0.29	-0.17

In numbers, the anomaly is strongest in standard CAPM with a very negative correlation of -0.82, one then sees how the beta effect is stepwise reduced by the various models. Both, FMAX and TMN factors are better in resolving the risk anomaly than the Fama & French (1993, 2015) SMB, HML, RMW, CWA factors. I further declare tail risk to be an even stronger driver of risk anomalies than lottery demand as stated by Frazzini & Pedersen (2014) since alpha-beta correlation is less under the CT model than under CF (see Table 22). Taking into account tail risk plus lottery demand, with CTF I do not only create a statistical high significant model, but also a framework that produces closest to zero alphas and importantly, a best in resolving the beta anomaly setting compared to other common asset pricing models.

6 Conclusion

After various tests, I find broad evidence for the existence of asset pricing's beta anomaly in the U.S. equity market by analyzing in total 3651 stocks over a time period of 2005 to 2018. No matter if empirical tests are built on a conditional or unconditional setting, the beta-anomaly can be verified in CAPM, Fama & French (1993)'s three factor and Fama & French (2015)'s five factor model. This result is in line with most literature such as Frazzini & Pedersen (2014) or Bali, Brown, Murray & Tang (2017). Furthermore, low beta stocks do not only outperform after risk-adjustment, but also in terms of absolute performance. Following Schneider et al. (2016), I introduce ex-ante measures on skewness and kurtosis to capture expectations about stock return distributions. Implementing ex-ante skewness and kurtosis directly into the Merton (1974) model, I show that this tail risk can be directly linked to credit risk, meaning that stocks with lower (more negative) skewness and higher kurtosis face greater credit risk than normal distributed opposites. By conditional testing using double sorts on rolling market beta and risk-neutral non-normality measures, ex-ante skewness positively predicts absolute and relative portfolio performance while the relation between ex-ante kurtosis and returns is negative, supporting statements of Schneider et al. (2016) and Bali, Hu & Murray (2017). As result, the higher the tail risk (aka credit risk), the lower are generated alphas. With this conclusion, the 'distress puzzle' cannot be resolved, but the empirical effects found are identical with the results of Campbell et al. (2008) or Schneider et al. (2016). Both, ex-ante skewness and kurtosis anomalies turn out to be highly significant and are confirmed by various robustness tests such as substituting market beta by option implied volatility or Frazzini & Pedersen (2014)' beta, but also by a random portfolio sampling method. To put this empirical survey into practical relevance, I set up performance back-tests for betting against beta (BaB) and betting against beta plus tail risk (BaB+TR) strategies. I find that BaB profitability can be statistically and economically increased when investors consider ex-ante skewness and kurtosis, which can be easily achieved without high effort. The economical importance of ex-ante tail risk is shown under both a beta neutral long-short strategy, but also for long only investors. Avoiding superior ex-ante tail risk in long-only portfolios turned out to generate a 10.4% higher excess return per annum. Comparing BaB with BaB+TR, the latter one evolved a 4.92 percentage points greater yearly excess return and higher Sharpe ratio.

Additionally, I give a short presentation of an alternative asset pricing model (CTF) consisting of market beta, Bali, Brown, Murray & Tang (2017)'s FMAX factor for lottery demand and a self developed factor TMN representing ex-ante tail risk. In direct comparison to CAPM and Fama & French (1993, 2015), the CTF model turns out to show equal goodness of fit, higher significance of regression coefficients and a better explanation of the beta anomaly. As final result, the absolute correlation between alpha and beta realized by the various models is reduced from 0.80 in CAPM over 0.62 at Fama & French (2015)'s five factor model down to 0.17 under the CTF model, hence CTF delivers the best explanation of the beta anomaly. Consequently, lottery demand and tail risk are the main drivers of the beta anomaly and accounting for them resolves it largely.

References

Alexander, D. & Veronesi, P. (2009), 'Macroeconomic uncertainty and fear measures extracted from index options', *manuscript* .

Altmann, E. (1968), 'Heteroskedasticity and autocorrelation consistent covariance matrix estimation', *Econometrica* **59**, p.817–858.

An, B.-J., Ang, A., Bali, T. & Cakici, N. (2014), 'The joint cross section of stocks and options', *Journal of Finance* **69**(5), p.2279–2337.

Ang, A., Hodrick, R., Xing, Y. & Zhang, X. (2006), 'The crosssection of volatility and expected returns', *Journal of Finance* **61**(1), p.259–299.

Bachelier, L. (1900), 'Theorie de la speculation', *Annales scientifiques de l'.N.S* **17**(3), p.21–86.

Baker, M., Bradley, B. & Taliaferro, R. (2014), 'The low-risk anomaly: A decomposition into micro and macro effects', *Financial Analysts Journal* **70**(2), p.43–58.

Baker, M., Bradley, B. & Wurgler, J. (2011), 'Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly', *Financial Analysts Journal Volume* **67**(1), p.40–54.

Baker, M., Wurgler, J. & Bradley, B. (2013), 'A behavioral finance explanation for the success of low volatility portfolios', *NYU Working Paper (accessed June 19, 2018)* **2451**(29537).
URL: *https://ssrn.com/abstract=2284643*

Bakshi, G., Kapadia, N. & Madan, D. (2003), 'Stock return characteristics, skew laws, and the differential pricing of individual equity options', *Review of Financial Studies* **16**(1), p.101–143.

Bali, T. (2009), 'Volatility spreads and expected stock returns', *Management Science* **55**(11), p.1797–1812.

Bali, T., Brown, S., Murray, S. & Tang, Y. (2014), 'Betting against beta or demand for lottery', *SSRN Electronic Journal (accessed June 19, 2018)* .
URL: *https://www.researchgate.net/publication/272299460*

Bali, T., Brown, S., Murray, S. & Tang, Y. (2017), 'A lottery-demand-based explanation of the beta anomaly', *Journal of Financial and Quantitative Analysis* **56**(6), p.2369–2397.

Bali, T., Cakici, N. & Whitelaw, R. (2011), 'Maxing out: Stocks as lotteries and the cross-section of expected returns', *Journal of Financial Economics* **99**(2), p.427–446.

Bali, T., Engele, R. & Murray, S. (2016), *Empirical Asset Pricing*, Wiley, chapter 7-8, pp. 103–145.

Bali, T., Hu, J. & Murray, S. (2017), 'Option implied volatility, skewness, and kurtosis and the cross-section of expected stock returns', *SSRN Electronic Journal (accessed June 19, 2018)* .
URL: *https://dx.doi.org/10.2139/ssrn.2322945*

Bali, T. & Murray, S. (2013), 'Does risk-neutral skewness predict the cross-section of equity option portfolio returns?', *Journal of Financial and Quantitative Analysis* **48**(4), p.1145–1171.

Bates, D. (2000), 'Post-'87 crash fears in the sp500 futures option market', *Journal of Econometrics* **94**, p.181–238.

Black, F. (1972), 'Capital market equilibrium with restricted borrowing', *The Journal of Business* **45**(3), p.444–455.

Black, F., Jensen, M. & Scholes, M. (1972), *The Capital Asset Pricing Model: Some Empirical Tests*, Praeger Publisher Inc., pp. p.79–121.

Black, F. & Scholes, M. (1973), ‘The pricing of options and corporate liabilitie’, *Journal of Political Economy* **81**(3), p.637–654.

Blitz, D. & van Vliet, P. (2007), ‘The volatility effect: Lower risk without lower return’, *Journal of Portfolio Management* **34**(1), p.102–113.

Blitz, D. & van Vliet, P. (2011), ‘Benchmarking low-volatility strategies’, *Journal of Index Investing* **2**(1), p.44–49.

Boehme, R., Danielsen, B., Kumar, P. & Sorescu, S. (2009), ‘Idiosyncratic risk and the cross-section of stock returns: Merton (1987) meets miller (1977)’, *Journal of Financial Markets* **12**(3), p.438–468.

Bogle, J. (2007), *The Little Book of Common Sense Investing*, Wiley.

Bollen, N. & Whaley, R. (2004), ‘Does net buying pressure affect the shape of implied volatility functions?’, *Journal of Finance* **59**(2), p.711–753.

Buss & Vilkov (2012), ‘Measuring equity risk with option-implied correlations’, *The Review of Financial Studies* **25**(10), p.3113–3140.

Campbell, J., Hilscher, J. & Szilagyi, J. (2008), ‘In search of distress risk’, *Journal of Finance* **63**(6), p.2899–2939.

Chabi-Yo, F. (2012), ‘Pricing kernels with stochastic skewness and volatility risk’, *Management Science* **58**(3), p.624–640.

Chang, B.-Y., Christoffersen, P., Jacobs, K. & Vainberg, G. (2012), ‘Option-implied measures of equity risk’, *Review of Finance* **16**(2), p.385–428.

Conrad, J., R.F., D. & Ghysels, E. (2013), ‘Ex ante skewness and expected stock returns’, *Journal of Finance* **68**(1), p.85–124.

Cremers, M., Driessen, J., Maenhout, P. & Weinbaum, D. (2008), ‘Individual stockoption prices and credit spreads’, *Journal of Banking and Finance* **32**, p.2706–2715.

Cremers, M. & Weinbaum, D. (2010), 'Deviations from put-call parity and stock return predictability', *Journal of Financial and Quantitative Analysis* **45**(2), p.335–367.

D'Avolio, G. (2002), 'The market for borrowing stock', *Journal of Financial Economics* **66**, p.271–306.

Demeterfi, K., Derman, E., Kamal, M. & Zou, J. (1999), 'A guide to volatility swaps', *Journal of Derivatives* **7**, p.9–32.

DeMiguel, V., Priyakha, Y., Uppal, R. & Vilkov, G. (2013), 'Improving portfolio selection using option-implied volatility and skewness', *Journal of Financial and Quantitative Analysis* **48**(6), p.1813–1845.

DeSantis, G. & Gerard, B. (1997), 'International asset pricing and portfolio diversification with time-varying risk', *Journal of Finance* **52**(5), p.1881–1912.

Dichev, I. (1998), 'Is the risk of bankruptcy a systematic risk?', *Journal of Finance* **53**, p.1131–1147.

Dimson, E. (1979), 'Risk measurement when shares are subject to infrequent trading', *Journal of Financial Economics* **7**(2), p.197–226.

Duan, J.-C. & Wei, J. (2009), 'Systematic risk and the price structure of individual equity options', *The Review of Financial Studies* **22**(5), p.1981–2006.

Dybvig, E. & Ross, S. (1985), 'Differential information and performance measurement using a security market line', *Journal of Finance* **40**(2), p.383–399.

Ecker, F. (2012), Asset pricing tests using random portfolios.

Falkenstein, E. (1994), Mutual Funds, Idiosyncratic Variance, and Asset Returns, PhD thesis, Northwestern University (accessed June 19, 2018).
URL: *https://www.researchgate.net/publication/269698051*

Fama, E. & French, K. (1993), 'Common risk factors in the returns on stocks and bonds', *Journal of Finacial Economics* **33**, p.3–56.

Fama, E. & French, K. (1996), 'Multifactor explanations of asset pricing anomalies', *Journal of Finace* **51**, p.55–84.

Fama, E. & French, K. (2015), 'A five-factor asset pricing model', *Journal of Finacial Economics* **116**(1), p.1–22.

Frazzini, A. & Pedersen, L. (2014), 'Betting against beta', *Journal of Finacial Economics* **111**(1), p.1–25.

French, D., Groth, J. & Kolari, J. (1983), 'Current investor expectations and better betas', *Journal of Portfolio Management* **10**(1), p.12–17.

Friend, I. & Westerfield, R. (1980), 'Coskewness and capital asset pricing', *Journal of Finance* **35**(4), p.897–913.

Garleanu, Pedersen & Poteshman (2009), 'Demand-based option pricing', *Review of Financial Studies* **22**(10), p.4259–4299.

Gibbson, M. (1982), 'Multivariate tests of financial models: a new approach', *Journal of Financial Economics* **10**, p.3–27.

Hagan, P. S., Kumar, D., Lesniewski, A. S. & E.Woodward, D. (2002), 'Managing smile risk', *Wilmott Magazine* **84**(3).

Han, B. (2008), 'Investor sentiment and options prices', *The Review of Financial Studies* **21**, p.387–414.

Harvey, C. & Siddique, A. (2000), 'Conditional skewness in asset pricing tests', *Journal of Finance* **55**(3), p.1263–1295.

Haugen, R. & Baker, N. (2012), 'Low risk stocks outperform within all observeable markets of the world', *SSRN Electronic Journal (accessed June 19, 2018)* .
URL: *https://dx.doi.org/10.2139/ssrn.2055431*

Haugen, R. & Heins, A. (1972), ‘On the evidence supporting the existence of risk premiums in the capital market’, *SSRN Electronic Journal (accessed June 19, 2018)* .
URL: *https://dx.doi.org/10.2139/ssrn.1783797*

Haugen, R. & Heins, A. (1975), ‘Risk and the rate of return on financial assets: Some old wine in new bottles.’, *Journal of Financial and Quantitative Analysis* **10**, p.775–784.

Jagannathan, R. & Wang, W. (1996), ‘The conditional capm and the cross-section of returns’, *Journal of Finance* **51**, p.3–53.

Jensen, M. (1968), ‘The performance of mutual funds in the period 1954 - 1964’, *Journal of Finance* **23**, p.389–416.

Johnson, T. (2005), ‘Forecast dispersion and the cross section of expected returns’, *Journal of Finance* **59**(5), p.1957–1978.

Kahnemann, D. & Tversky, A. (1979), ‘Prospect theory: An analysis of decision under risk’, *Econometrica* **42**(2), p.263–292.

Kale, J. & Sheth, A. (2015), ‘Power-log portfolio optimization for managing downside risk’.

Kandel, S. (1984), ‘The likelihood ratio test statistic of mean-variance efficiency without a riskless asset’, *Journal of Financial Economics* **13**, p.575–592.

Kane (1982), ‘Skewness preference and portfolio choice’, *Journal of Financial and Quantitative Analysis* **17**(1), p.15–25.

Kelly, B. & Jiang, H. (2014), ‘Tail risk and asset prices’, *Review of Finacial Studies* **27**(10), p.2841–2871.

Konno, H., Shirakawa, H. & Yamazaki, H. (1993), ‘A mean-absolute deviation-skewness portfolio optimization model', *Annals of Operations Research* **45**(1), p.205–220.

Kraus, A. & Litzenberger, R. (1976), 'Skewness preference and the valuation of risk assets', *Journal of Finance* **31**, p.1085–1100.

Lewellen, J. & Nagel, S. (2006), 'The conditional capm does not explain asset-pricing anomalies', *Journal of Financial Economics* **82**, p.289–314.

Lewellen, J., Nagel, S. & Shanken, J. (2010), 'A skeptical appraisal of asset pricing tests', *Journal of Financial Economics* **96**, p.175–194.

Li, X., Sullivan, R. & Garcia-Feijo, L. (2014), 'The limits to arbitrage and the low-volatility anomaly', *Financial Analysis Journal* **70**(1), p.52–64.

Lin, T.-C. & Liu, X. (2017), 'Skewness, individual investor preference, and the cross-section of stock returns', *Review of Finance (accessed June 19, 2018)* p. rfx036.
URL: *http://dx.doi.org/10.1093/rof/rfx036*

Lintner, J. (1965), 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', *Review of Economics and Statistics* **47**, p.13–37.

Liu, Z. & van der Heijden, T. (2015), 'Model-free risk-neutral moments and proxies', *SSRN Electronic Journal (accessed June 19, 2018)* .
URL: *http://dx.doi.org/10.2139/ssrn.2641559*

Lo, A. & MacKinlay, A. (1990), 'Data-snooping biases in tests of financial asset pricing models', *Review of Financial Studies* **3**(3), p.431–467.

Markowitz, H. (1952), 'Portfolio selection', *Journal of Finance* **7**(1), p.77–91.

Mayhew, S. (2002), 'Risk-neutral skewness: Evidence from stock options', *Journal of Financial and Quantitative Analysis* **37**(3), p.471–493.

Merton (1980), ‘On estimating the expected return on the market: An exploratory investigation’, *Journal of Financial Economics* **8**(4), p.323–361.

Merton, R. (1974), ‘On the pricing of corporate debt: the risk structure of interest rates’, *Journal of Finance* **29**(2), p.449–470.

Mixon, S. (2010), ‘What does implied volatility skew measure?’, *The Journal of Derivatives* **18**(4), p.9–25.

Mossin, J. (1966), ‘Equilibrium in a capital asset market’, *Econometrica* **35**, p.768–783.

Ohlson, J. (1980), ‘Financial ratios and the probabilistic prediction of bankruptcy’, *Journal of Accounting Research* **18**, p.109–131.

Rehmann, Z. & Vilkov, G. (2012), ‘Risk-neutral skewness: Return predictability and its sources’, *SSRN Electronic Journal (accessed June 19, 2018)* .
URL: *https://dx.doi.org/10.2139/ssrn.1301648*

Rubinstein, M. (1973), ‘The fundamental theorem of parameter-preference security valuation’, *Journal of Finacial and Quantitative Analysis* **8**, p.61–69.

Russell, G.-A., Impson, C. & Karafiath, I. (1994), ‘An empirical investigation of beta stability: Portfolios vs. individual securities’, *Journal of Business, Finance and Accounting* **21**(6), p.909–916.

Schneider, P., Wagner, C. & Zechner, J. (2016), ‘Low risk anomalies?’, *CFS Working Paper, No.550 (accessed June 19, 2018)* .
URL: *https://dx.doi.org/10.2139/ssrn.2858933*

Scholes, W. (1977), ‘Estimating betas from nonsynchronous data’, *Journal of Financial Economics* **5**(3), p.309–327.

Shanken, J. (1985), ‘Multivariate tests of the zero-beta capm’, *Journal of Financial Economics* **14**, p.327–348.

Sharpe, W. (1964), ‘Capital asset prices: A theory of market equilibrium under conditions of risk’, *American Finance Association* **19**(3), p.425–442.

Vasicek, O. (1973), ‘A note on using cross-sectional information in bayesian estimation of security betas’, *Journal of Finance* **28**(5), p.1233–1239.

Xing, Y. (2010), ‘What does the individual option volatility smirktell us about future equity returns?’, *Journal of Financial and Quantitative Analysis* **45**(3), p. 641–662.

A Initial Empirical Tests

Applying random portfolio bootstrapping with 1000 samples, tested under CAPM with the own market portfolio formed out of the available stock universe indicates positive out-performance of low-beta stocks compared to high beta side (Fig. 24).

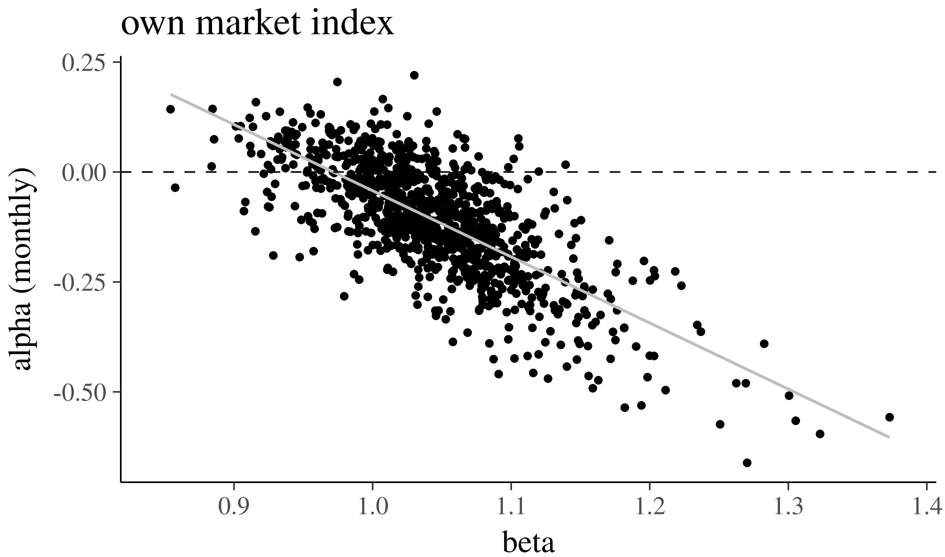

Figure (24) Bootstrapping 1000 random portfolios with own market index as benchmark, tested in CAPM.

Table (23) Summary statistics of the bootstrapping using own market index and CAPM

	N	Mean	St. Dev.	Min	Max
alpha	1,000	−0.1094	0.1303	−0.6614	0.2199
p-value α	1,000	0.3677	0.2919	0.0035	0.9989
beta	1,000	1.0444	0.0633	0.8542	1.3731
p-value β	1,000	0.0000	0.0000	0	0
R^2	1,000	0.9452	0.0280	0.7752	0.9804

B Using Frazzini & Pedersen (2014)'s Beta

Figure (25) Beta anomaly using Frazzini & Pedersen (2014)'s estimation of the systematic risk coefficient

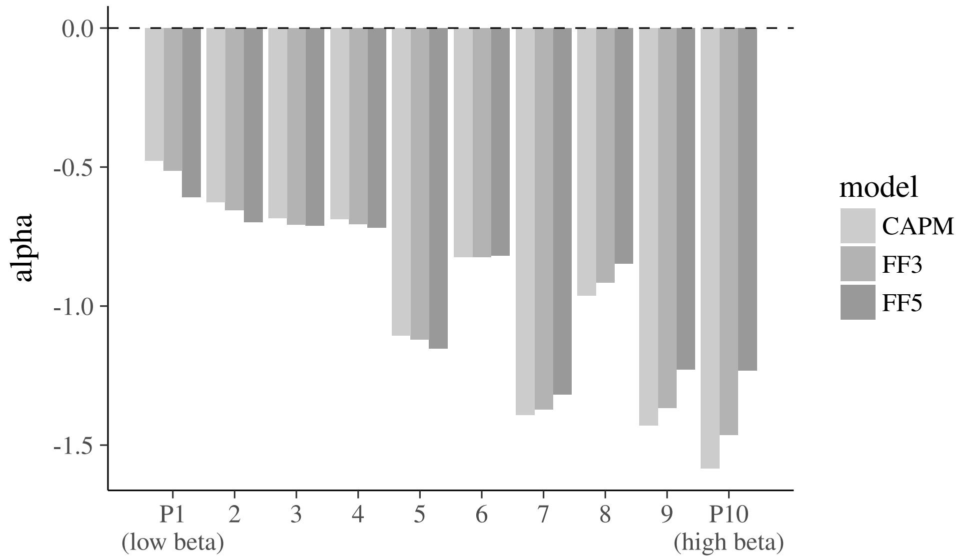

Table (24) Portfolios sorted on Frazzini & Pedersen (2014)'beta estimation

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
excess return	0.07	0.02	0.02	0.07	-0.3	0.03	-0.48	0.03	-0.3	-0.3
CAPM alpha	**-0.48**	**-0.63**	**-0.68**	**-0.69**	**-1.11**	**-0.82**	**-1.39**	**-0.96**	**-1.43**	**-1.58**
	(-4.28)	(-5.8)	(-5.11)	(-6.15)	(-3.81)	(-4.84)	(-8.21)	(-5.67)	(-6.43)	(-4.63)
FF3 alpha	**-0.51**	**-0.65**	**-0.71**	**-0.71**	**-1.12**	**-0.82**	**-1.37**	**-0.92**	**-1.37**	**-1.46**
	(-4.28)	(-5.8)	(-5.11)	(-6.15)	(-3.81)	(-4.84)	(-8.21)	(-5.67)	(-6.43)	(-4.63)
FF5 alpha	**-0.61**	**-0.7**	**-0.71**	**-0.72**	**-1.15**	**-0.82**	**-1.32**	**-0.85**	**-1.23**	**-1.23**
	(-5.33)	(-6.23)	(-5.13)	(-6.27)	(-3.91)	(-4.8)	(-7.95)	(-5.32)	(-6.02)	(-4.14)
beta (ex-ante)	0.54	0.62	0.66	0.7	0.74	0.78	0.82	0.87	0.95	1.33
beta (real.)	0.79	0.89	0.96	1.03	1.08	1.12	1.16	1.22	1.38	1.5
volatility	14.81	16.82	18.51	19.57	23.79	22.59	23.95	25.95	29.94	35.6
skewness (real.)	-0.15	-0.37	-0.75	-0.33	-6.42	0.1	-0.73	-0.28	-0.6	-0.56
kurtosis (real.)	17.52	13.68	11.41	12.07	172.66	14.87	7.48	7.67	10.9	12.87

significant alpha values (p-value $< 5\%$) are displayed in bold

C Trading and Short-Selling Cost Estimation

Table (25) Estimation of transaction costs for the BaB and BaB+TR strategies, in percent monthly

	BaB			*BaB+TR*		
	PF	long	short	PF	long	short
short selling costs	0.017		0.021	0.012		0.021
trading costs	0.002	0.001	0.001	0.037	0.032	0.014
net excess return	0.86	0.47	-0.05	1.24	0.56	0.34
net CAPM alpha	0.81	-0.07	-1.28	1.27	0.05	-1.81
net FF3 alpha	0.71	-0.10	-1.20	1.12	0.02	-1.64
net FF5 alpha	0.41	-0.17	-0.97	0.82	-0.06	-1.34
beta (realized)	0.06	0.71	1.6	-0.04	0.65	1.81
volatility	14.6	14.16	32.11	20.6	14.5	39.61

D Tests on TMN and FMAX

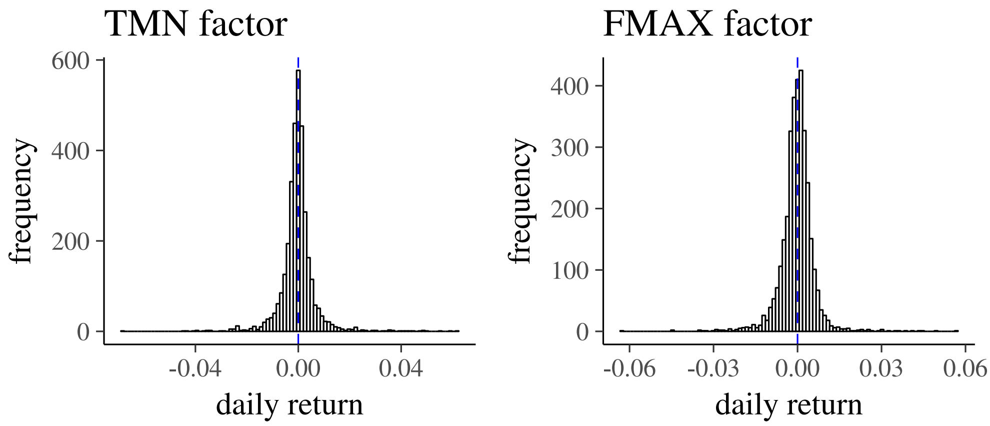

Figure (26) Return distribution of the TMN and FMAX factors

Table (26) One sample t-test of TMN and FMAX indicate that both factors are significantly negative

	TMN	FMAX
H0	$\mu = 0$	$\mu = 0$
H1	$\mu < 0$	$\mu < 0$
mean [% p.a.]	-5.66	-5.10
t	-1.85	-1.89
p-value	0.032	0.030
95% confidence interval	-Inf to -0.62	-Inf to -0.66

E Residuals of CTF model

This appendix section deals with the inspection of the residuals realized under CTF model when testing the ex-ante beta sorted portfolios. P1 denotes the low beta portfolio, P5 mid beta and P10 high beta. All residuals seem to fit a normal distribution with slightly higher kurtosis (cp. 27), thus from this side I do not see any violations for linear regression assumptions.

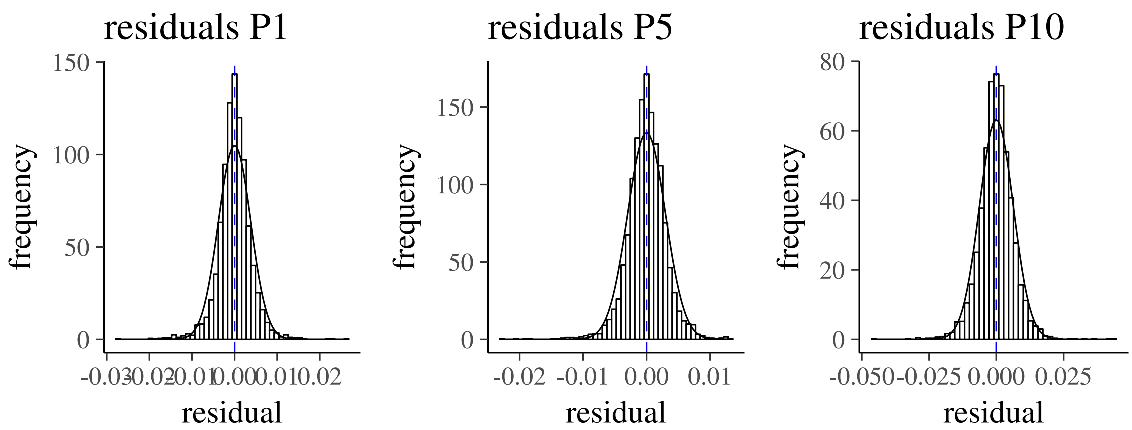

Figure (27) Residual distribution of the CTF model on beta sorted portfolios

When plotting the residuals over time (see Fig. 28), higher variance after the financial crises in 2008 is observable. But besides, I do not find systematic structures of heteroscedasticity. Thus, also from this side I do not see any sever violations and believe CTF's homoskedasticity to generally hold under common financial times.

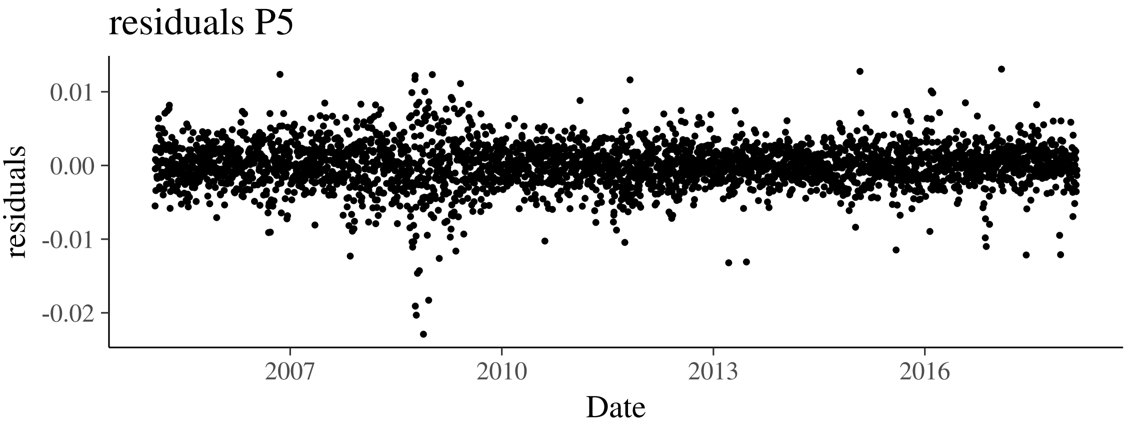

Figure (28) Residuals of P5 (mid beta) over time tested in the CTF model; Visual inspection for heteroscedasticity.

Testing for coeffiecient's correlation, I find that the predictors do not show strong multicollinearity allowing the model to be assumed unbiased. Given that each predictor's variance inflation factor (VIF) is close to one, I see multicollinearity to be very low. VIF of exactly 1 would mean that the coefficient is uncorrelated, VIF values above 4 would create concerns about multicollinearity. With the tolerance I express the percentage of predictor's variance that cannot be accounted for by other predictors, also here I find sounding robustness. Collinearity diagnostic upon eigenvalues does not indicate any misspecification either, as the first Eigenvalue is above 2 but all following ones are below.

Table (27) Analysis of predictor's multicollinearity

	Eigenvalue	Condition Index	intercept	beta	TMN	FMAX
	2.32	1.00	0.00	0.07	0.06	0.06
	1.00	1.52	0.98	0.00	0.00	0.00
	0.42	2.36	0.02	0.84	0.32	0.04
	0.26	2.98	0.00	0.09	0.61	0.90
tolerance				0.543	0.445	0.392
VIF				1.84	2.25	2.55

Using the Durbin-Watson test allows to compare autocorrelation of residuals within each model applied on beta sorted portfolios. Within every model, the test statistic is close to 2 ranging from 1.79 to 2.11. Therefore, as no value largely deviates from 2 I do not detect any strong violation in model specification by autocorrelated residuals.

Table (28) Durbin Watson test on beta-sorted portfolios

	P1 (low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high beta)
CAPM	1.96	2.06	2.01	2.06	$1.95^{\cdot}$	1.92^{*}	1.82^{***}	2.02	2.07	2.11
FF3	1.91^{**}	2.03	$1.95^{\cdot}$	1.97	1.93^{*}	1.92^{*}	1.82^{***}	1.98	1.99	2
FF5	1.85^{***}	2.01	1.92^{*}	1.97	1.93^{*}	1.91^{**}	1.81^{***}	1.96	1.9^{**}	1.98
CT	1.87^{***}	2.02	1.99	2.01	1.93^{*}	1.9^{**}	1.81^{***}	1.98	1.88^{***}	$1.94^{\cdot}$
CF	1.92^{*}	2	1.93^{*}	2.02	$1.95^{\cdot}$	1.93^{*}	1.79^{***}	1.95	1.9^{**}	1.99
CTF	1.89^{**}	2.01	1.97	2.01	1.93^{*}	1.92^{*}	1.79^{***}	1.95	1.85^{***}	1.96

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$, $^{\cdot}p < 0.1$